

# Schubert cells on $Gr_2(\mathbb{P}^1, \mathbb{P}^3)$

$L$	$\dim L \cap \langle e_3 \rangle$	$\dim L \cap \langle e_2, e_3 \rangle$	$\dim L \cap \langle e_1, e_2, e_3 \rangle$	VECTOR	$(\lambda_1, \lambda_2)$
$\begin{bmatrix} 1 & * & * \\ & 1 & * & * \end{bmatrix}$	$\geq 0$	$\geq 0$	$\geq 1$	$(0,0,0)$	$(0,0)$
$\begin{bmatrix} 1 & * & * \\ & 1 & * \end{bmatrix}$	$\geq 0$	$\geq 1$	$\geq 1$	$(0,1,0)$ $L \cap F_2 \geq 1$	$(1,0)$ $\square$
$\begin{bmatrix} 1 & * & * \\ & & & 1 \end{bmatrix}$	$\geq 1$	$\geq 1$	$\geq 1$	$(1,1,0)$ $L \cap F_3 \geq 1$	$(2,0)$ $\square$
$\begin{bmatrix} 0 & 1 & * \\ & & 1 & * \end{bmatrix}$	$\geq 0$	$\geq 1$	$\geq 2$	$(0,1,1)$ $L \cap F_3 \geq 2$	$(1,1)$ $\square$
$\begin{bmatrix} 0 & 1 & * \\ & & & 1 \end{bmatrix}$	$\geq 1$	$\geq 1$	$\geq 2$	$(1,1,1)$ $L \cap F_3 \geq 2$	$(2,1)$ $\square$
$\begin{bmatrix} 0 & 0 & 1 \\ & & & 1 \end{bmatrix}$	$\geq 1$	$\geq 2$	$\geq 2$	$L \cap F_3 \geq 1$ $(1,2,1)$ $L \cap F_2 \geq 2$	$(2,2)$ $\square$

$0 = F_0$   
 $\langle e_3 \rangle = F_1$   
 $\langle e_2, e_3 \rangle = F_2$   
 $\langle e_1, e_2, e_3 \rangle = F_3$   
 $\langle e_0, \dots, e_3 \rangle = F_4 = V$

$$\dim L \cap F_{2+i-\lambda_i} \geq i \quad i=1,2$$

$\lambda_1 = \text{FIRST } j$   
 s.t.  
 $\dim L \cap F_{3-j} \geq 1$

$\lambda_2 = \text{FIRST } j$   
 s.t.  
 $\dim L \cap F_{4-j} \geq 2$

The open part (dense cell)  
is given by

$$\{L \mid L \cap \langle e_2, e_3 \rangle = \emptyset\}$$

↑ empty in  
projective space

and for general  $Gr(P^k, P^n)$  is:

$$\{L \mid L \cap F_{n-k} = \emptyset\}$$

When equality holds in the table  
of previous page we have  
the open cell.

The condition with  $\geq$   
gives the projective closure.