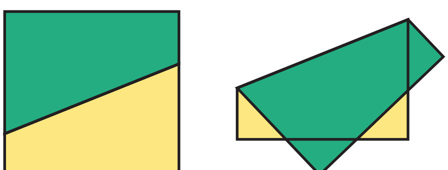
About the Cover

Fractal Origami

The cover image is a variant of Figure 12 in this issue's article by Bernard Dacorogna, Paolo Marcellini, and Emanuele Paolini on some mathematical aspects of origami.

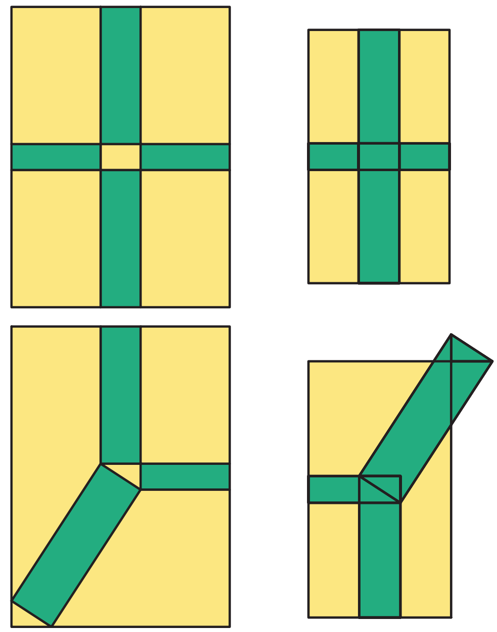
The connection between the image and origami might not be apparent. A partition of a planar region Ω into any number of convex polygons is said to satisfy the *Kawasaki angle condition* if at every vertex there is an even number of polygons meeting at each vertex, and the alternating sum of angles there vanishes. A partition satisfying this condition determines a map from Ω into the plane with the property that maps on neighboring polyhedra are obtained by reflection. The Kawasaki condition implies that this specification is consistent. I call such maps **fold maps**. The simplest nontrivial fold map is the basic origami move.



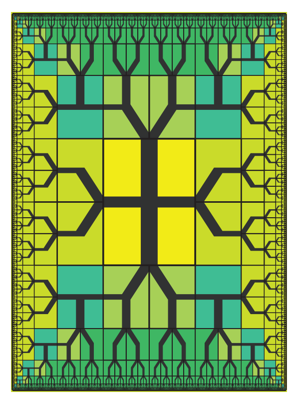
In this and other figures, I follow Dacorogna et al. in distinguishing those domains that change orientation from those that don't.

A fold map F is contracting in the sense that $|F(P) - F(Q)| \leq |P - Q|$. A curious question arises: *given a domain Ω and a boundary path map Φ from its boundary $\partial\Omega$ satisfying some contraction condition, does there exist a partition whose associated fold map agrees with Φ on the boundary?* According to Dacorogna et al., this is so if the boundary map satisfies the strict contraction condition $|F(P) - F(Q)| \leq \lambda|P - Q|$ for some $\lambda < 1$. Any map satisfying other than a very simple boundary constraint must come from an infinite partition, and one might expect that the behaviour of the partition near the boundary will be interesting. In Figure 12 of their article, and on the cover of this issue, are illustrated partitions that give rise to a linear boundary map in the case where Ω is a rectangle.

This seems to be the only large class with an explicit construction. In both these cases, the partition is by rectangles of different sizes but of only two basic designs. It is easy to draw the unadorned picture without much trouble. It is made up of rectangles of various sizes, but up to similarity exactly the two basic designs shown above, which Dacrorogna et al. call the *base regions*: It is relatively easy to draw the figure in a straightforward way, since the pattern of rectangles is easy to lay out, and the region at the top occurs precisely on diagonals. But it is somewhat more interesting to do something that



reflects more directly the recursive branching structure. The regions can be assembled in a directed acyclic graph with the central region as root and edges going out from the center to smaller and smaller regions. This can be pruned to become a tree, one associated to a finite state diagram of five states, marked by distinct colors in this figure:



Using this finite state machine to traverse the tree makes possible what might be called "intelligent" drawing. *What other boundary conditions on fold maps can be realized by finite state diagrams? What can one say in general about the relationship between a boundary condition and the structure of a partition whose fold map satisfies it?*

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