

Poiché $\Sigma_2 \subseteq \{(x, y, z) : z = 4 - 2(x+y)\}$ si ha (7)

$$\nu_{\text{ext}}|_{\Sigma_2} = \frac{(2, 2, 1)}{\|(2, 2, 1)\|} = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right).$$

Quindi:

$$\begin{aligned} \int_{\Sigma_1} \vec{F} \cdot \nu_{\text{ext}} dV &= - \int_{\Sigma_2} \vec{F} \cdot \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) dV \\ &= - \iint_D (-2x - 2y^3 - 2(4 - 2(x+y))^2 + 2y - x^2y) dx dy \\ &= 4(4 + x^2 + 2xy + y^2 - 4x - 4y) \end{aligned}$$

Passando a coordinate polari: $x+1 = \rho \cos \theta$,
 $y+1 = \rho \sin \theta$, con $(\rho, \theta) \in [0, \sqrt{5}] \times [0, 2\pi]$, si ottiene

$$\begin{aligned} \int_{\Sigma_1} \vec{F} \cdot \nu_{\text{ext}} dV &= \int_0^{2\pi} d\theta \int_0^{\sqrt{5}} \rho \left[30(\rho^{\cos \theta} - 1) - 2(\rho^{\sin \theta} - 1)^3 - 32 + \right. \\ &\quad \left. - 8((\rho^{\cos \theta} - 1)^2 + (\rho^{\sin \theta} - 1)^2) - 16(\rho^{\cos \theta} - 1)(\rho^{\sin \theta} - 1) + 34(\rho^{\sin \theta} - 1) \right. \\ &\quad \left. - (\rho^{\cos \theta} - 1)^2(\rho^{\sin \theta} - 1) \right] d\rho \end{aligned}$$

Usando la simmetria e periodicità di $\theta \rightarrow \cos^k \theta$,
 $\theta \rightarrow \sin^k \theta$, k dispari, si ottiene

$$\begin{aligned} \int_{\Sigma_1} \vec{F} \cdot \nu_{\text{ext}} dV &= \int_0^{2\pi} d\theta \int_0^{\sqrt{5}} \rho \left[-125 + 2\rho^2 \sin^2 \theta - 8\rho^2 + \rho^2 \cos^2 \theta \right] d\rho \\ &= -125\pi \left[\rho^2 \right]_0^{\sqrt{5}} + 2\pi \left[\frac{\rho^4}{4} \right]_0^{\sqrt{5}} - 16\pi \left[\frac{\rho^3}{3} \right]_0^{\sqrt{5}} + \\ &\quad + \pi \left[\frac{\rho^4}{4} \right]_0^{\sqrt{5}} \\ &= -867\pi \end{aligned}$$