

Per semplificare i conti merita di (3) maggiorazioni/minorazioni su φ dato che determinarne direttamente il max è complicato. Distinguiamo due casi:

$t \in [0, 1/2]$: $\frac{1}{2} \leq 1-t \leq 1$, da cui

$$\frac{1}{2} \frac{t^\beta}{\delta^2 + t} \leq \varphi(t) \leq \frac{t^\beta}{\frac{\delta^2}{2} + t}$$

Si noti che

$$0 \leq \frac{d}{dt} \left(\frac{t^\beta}{\delta^2 + t} \right) = \frac{t^{\beta-1}}{(\delta^2 + t)^2} (\beta(\delta^2 + t) - t) \Leftrightarrow$$

$$(\beta-1)t + \beta\delta^2 \geq 0$$

$$\Rightarrow \frac{2^{-\beta-1}}{\delta^2 + \frac{1}{2}} \leq \max_{[0, 1/2]} \varphi \leq \frac{2^{-\beta}}{\frac{\delta^2}{2} + \frac{1}{2}} \quad \beta \geq 1$$

$$\frac{1}{2} \frac{\left(\frac{\beta}{1-\beta} \delta^2 \right)^\beta}{\delta^2 + \frac{\beta}{1-\beta} \delta^2} \leq \max_{[0, 1/2]} \varphi \leq \frac{\left(\frac{\beta}{1-\beta} \frac{\delta^2}{2} \right)^\beta}{\frac{\beta}{1-\beta} \frac{\delta^2}{2} + \frac{\delta^2}{2}} \quad \beta < 1$$

$$= \frac{1}{2} \frac{\beta^\beta}{(1-\beta)^{\beta-1}} \delta^{2\beta-2} = C(\beta)$$

$$= \frac{\beta^\beta}{[k \cdot (1-\beta)]^{\beta-1}} \delta^{2\beta-2} = C'(\beta)$$

$t \in [1/2, 1]$:

$$\frac{2^{-\beta}(1-t)}{\delta^2(1-t)^2 + 1} \leq \varphi(t) \leq \frac{1-t}{\delta^2(1-t)^2 + \frac{1}{2}}$$

da cui:

$$\frac{2^{-\beta-1}}{\frac{\delta^2}{2} + 1} \leq \max_{[1/2, 1]} \varphi \leq \frac{1}{\frac{\delta^2}{2} + 1}$$