$\boxed{\text{ESERCIZIO 2:}}$ $\omega : \mathbb{R}^2 \setminus \{(0,0)\} \longrightarrow (\mathbb{R}^2)^*$ di classe $C^{(1)}$ chiusa t.c.

$$\lim_{\|(x,y)\| \to +\infty} \left(\omega_1(x,y) \Big/ -\frac{y}{x^2+y^2} \right) = \lim_{\|(x,y)\| \to +\infty} \left(\omega_2(x,y) \Big/ \frac{x}{x^2+y^2} \right) = 1$$

e γ una curva regolare semplice e chiusa che circonda l'origine orientata in senso antiorario: $\gamma : [a,b] \longrightarrow \mathbb{R}^2 \setminus \{(0,0)\}$.
Sia $R_0 > 0$ t.c. $\gamma([a,b]) \subsetneq \{(x,y) : x^2+y^2 \leq R^2\} = C_R$, $\forall R \geq R_0$.
Dalle formule di Gauss-Green si ha

$$\int_\gamma \omega = \int_{\partial^+ C_R} \omega = \int_{\partial^+ C_R} \left(\omega_1(x,y) + \frac{y}{x^2+y^2} \right) dx + \left(\omega_2(x,y) - \frac{x}{x^2+y^2} \right) dy$$

$$+ \underbrace{\int_{\partial^+ C_R} \left(-\frac{y}{x^2+y^2} dx + \frac{x}{x^2+y^2} dy \right)}_{= 2\pi}$$

D'altra parte,

$$\int_{\partial^+ C_R} \left(\omega_1(x,y) + \frac{y}{x^2+y^2} \right) dx = \int_{\partial^+ C_R} \frac{-y}{x^2+y^2} \left(\frac{\omega_1(x,y)}{\frac{-y}{x^2+y^2}} - 1 \right) dx$$

$$= \int_0^{2\pi} -\frac{R}{R^2} \operatorname{sen}\theta \left(\frac{\omega_1(R\cos\theta, R\operatorname{sen}\theta)}{-\frac{\operatorname{sen}\theta}{R}} - 1 \right) (-R\operatorname{sen}\theta) \, d\theta$$

$$= \int_0^{2\pi} + \operatorname{sen}^2\theta \left(\frac{\omega_1(R\cos\theta, R\operatorname{sen}\theta)}{-\frac{\operatorname{sen}\theta}{R}} - 1 \right) d\theta.$$