

si ottiene

(9)

$$I = \iiint_D 4z \, dx \, dy \, dz - \iint_{\Sigma_1} \vec{F} \cdot (-1, 0, 0) \, dy \, dz +$$

$$- \iint_{\Sigma_2} \vec{F} \cdot (0, -1, 0) \, dx \, dz - \iint_{\Sigma_3} \vec{F} \cdot (0, 0, -1) \, dx \, dy \, dz.$$

D'altra parte, si ha

$$(x, y, z) \in \Sigma_1 \Rightarrow \vec{F}(x, y, z) \cdot (-1, 0, 0) = 0$$

$$(x, y, z) \in \Sigma_2 \Rightarrow \vec{F}(x, y, z) \cdot (0, -1, 0) = 0$$

$$(x, y, z) \in \Sigma_3 \Rightarrow \vec{F}(x, y, z) \cdot (0, 0, -1) = 0$$

e quindi

$$I = \iiint_D 4z \, dx \, dy \, dz = \iint_{D'} dx \, dy \int_0^{\sqrt{4-4x^2-4y^2}} 4z \, dz$$

$$D' = \{(x, y) \in [0, \pi]^2 : 4x^2 + 4y^2 \leq 4\}$$

$$= 2 \iint_{D'} (4 - 4x^2 - 4y^2) \, dx \, dy = 4 \int_0^{2\pi} d\theta \int_0^1 (4 - 4s^2) s \, ds$$

$$\begin{cases} x = s \cos \theta \\ y = s \sin \theta \end{cases}$$

$$= 32\pi \int_0^1 (s - s^3) \, ds = 32\pi \left( \frac{1}{2} - \frac{1}{4} \right) = 8\pi.$$