

Sia

(7)

$$I = \int_{\Sigma} (x^2 + y^2)^{-3/2} (4(x^2 + y^2) + 2)^{-1/2} dV$$

allora

$$I = \int_1^2 dz \int_0^{2\pi} (\cancel{f(\theta)})^{-3} (4\cancel{f^2(\theta)} + 2)^{-1/2} \frac{(3 + |\cancel{f(2\theta)}|)^{1/2}}{(\cancel{f(\theta)})^{-3}} d\theta$$

$$\frac{4}{1 + |\sin(2\theta)|} + 2 = 2 \frac{3 + |\sin(2\theta)|}{1 + |\sin(2\theta)|}$$

$$= \int_0^{2\pi} \frac{1}{\sqrt{2}} \sqrt{1 + |\sin(2\theta)|} d\theta = \sqrt{2} \int_0^{\pi} \sqrt{1 + \sin\theta} d\theta$$

$$\stackrel{\uparrow}{=} \sqrt{2} \int_{t=\tan(\theta/2)}^{+\infty} \underbrace{\sqrt{1 + \frac{2t}{1+t^2}}}_{=2} \frac{2}{1+t^2} dt$$

$$= 2 \frac{1+t}{(1+t^2)^{3/2}}$$

D'altra parte:

$$\int \frac{t}{(1+t^2)^{3/2}} dt = -\frac{1}{\sqrt{1+t^2}} + C$$

integrando
per parti con
 $f(t)=t, g'(t)=\frac{t}{(1+t^2)^{3/2}}$

$$\int \frac{1}{(1+t^2)^{3/2}} dt \stackrel{\uparrow}{=} \int \frac{1}{\sqrt{1+t^2}} dt - \int \frac{t^2}{(1+t^2)^{3/2}} dt$$

$$\int \frac{1}{\sqrt{1+t^2}} dt - \left(-\frac{t}{\sqrt{1+t^2}} + \int \frac{1}{\sqrt{1+t^2}} dt \right) = \frac{t}{\sqrt{1+t^2}} + C$$

Infine:

$$I = 2\sqrt{2} \left[\frac{t-1}{\sqrt{1+t^2}} \right]_0^{+\infty} = 2\sqrt{2} [1 - (-1)]$$

$$= 4\sqrt{2}$$