$\boxed{\text{ESERCIZIO 2:}}$ La forma differenziale

$$\omega(xyz) = \mathrm{Ch}^3(xy)\,dx + \frac{6y^5 - 3y^2z^2}{y^6+z^4}\,dy + \frac{2y^3z + 4z^3}{y^6+z^4}\,dz$$

risulta di classe $C^1(\mathbb{R}^3 \setminus \{(xyz) : y^2+z^2 = 0\})$.
Inoltre, se

$$\omega_1(xyz) = \mathrm{Ch}^3(xy)\,dx - \frac{3y^2z^2}{y^6+z^4}\,dy + \frac{2y^3z}{y^6+z^4}\,dz$$

si trova che

$$\omega = \omega_1 + d\left(\ln(y^6+z^4)\right).$$

Poiché la curva γ $\quad x=2,\ y^2+z^2=1$ è piana si ottiene

$$\int_\gamma \omega = \int_\gamma \omega_1 = \int_\gamma \Big(\underbrace{-\frac{3y^2z^2}{y^6+z^4}\,dy + \frac{2y^3z}{y^6+z^4}\,dz}_{\omega_2(x,y,z)}\Big)$$

$$= \int_{\gamma'} \omega_2(2,y,z)$$

dove $\overbrace{\omega_2(2,y,z)}^{\omega_3(y,z)} : \mathbb{R}^2 \setminus \{y^2+z^2=0\} \longrightarrow (\mathbb{R}^2)^*$ e γ' è la curva $y^2+z^2=1$ orientata in senso antiorario.
La forma ω_3 è chiusa:

$$\frac{\partial}{\partial z}\left(-\frac{3y^2z^2}{y^6+z^4}\right) = \frac{-3y^2}{(y^6+z^4)^2}\left(2z(y^6+z^4) - z^2\cdot 4z^3\right)$$

$$= \frac{-3y^2}{(y^6+z^4)^2}\left(2zy^6 - 2z^5\right)$$

$$\frac{\partial}{\partial y}\left(\frac{2y^3z}{y^6+z^4}\right) = \frac{2z}{(y^6+z^4)^2}\left(3y^2(y^6+z^4) - y^3\cdot 6y^5\right)$$

$$= \frac{2z}{(y^6+z^4)^2}\left(3y^2z^4 - 3y^8\right)$$