$\boxed{\text{ESERCIZIO 4:}}$ Sia $\Psi(x,y)=\left(y-\frac{x}{2},\frac{y}{x}\right)$, e siano (7)

$$U=\{(x,y)\in\mathbb{R}^2:\ x>0,\ y>x/2\},$$
$$V=\{(\xi,\eta)\in\mathbb{R}^2:\ \xi>0,\ \eta>1/2\}.$$

Proviamo che $\Psi: U\longrightarrow V$ è biunivoca. Infatti

$$\begin{cases}\xi=y-\frac{x}{2}=\left(\frac{y}{x}-\frac{1}{2}\right)x=\left(\eta-\frac{1}{2}\right)x\\ \eta=\frac{y}{x}\end{cases}$$

poiché $V\subset\{\eta>\frac{1}{2}\}$ si ha allora

$$\begin{cases}x=\dfrac{\xi}{\eta-\frac{1}{2}}\\ y=\dfrac{\eta\xi}{\eta-\frac{1}{2}}\end{cases}.$$

Infine $x>0$ e $y>\frac{x}{2}\Rightarrow\eta=\frac{y}{x}>\frac{1}{2}$, e poiché $\xi=\eta x$, $\xi>0$.

Sia $\Phi(\xi,\eta)=\left(\dfrac{\xi}{\eta-\frac{1}{2}},\dfrac{\xi\eta}{\eta-\frac{1}{2}}\right)$, allora $\Phi\in C^1(V,U)$, e

$$\det\nabla\Phi(\xi,\eta)=\det\begin{pmatrix}\dfrac{1}{\eta-\frac{1}{2}} & \dfrac{\eta}{\eta-\frac{1}{2}}\\ -\dfrac{\xi}{(\eta-\frac{1}{2})^2} & -\dfrac{\xi}{2(\eta-\frac{1}{2})^2}\end{pmatrix}$$

$$=-\frac{\xi}{(\eta-\frac{1}{2})^2}$$

quindi Φ risulta un cambiamento ammissibile di coordinate fra U e V.