

$$\begin{aligned}
 \sum_{m \geq 1} \frac{1}{m} x^{2m} &= 2 \sum_{m \geq 1} \frac{x^{2m}}{2m} = 2 \sum_{m \geq 1} \int_0^x t^{2m-1} dt \stackrel{|x| < 1}{=} \\
 2 \int_0^x \sum_{m \geq 1} t^{2m-1} dt &= 2 \int_0^x \frac{1}{t} \sum_{m \geq 1} t^{2m} dt = \\
 2 \int_0^x \frac{1}{t} \left(\frac{1}{1-t^2} - 1 \right) dt &= 2 \int_0^x \frac{t}{1-t^2} dt = \\
 -\ln(1-x^2)
 \end{aligned}$$

Conclusion

$$S(x) = \sum_{m \geq 1} \left(m^2 - \frac{1}{m} \right) x^{2m} = \frac{x^2(x^2+1)}{(1-x^2)^3} - \ln(1-x^2)$$