Resta quindi solo da calcolare:

$$\iint_E \left(4-(x^2+y^2)\right)(y^2-x^2)\,dx\,dy = \begin{cases} x = \frac{2}{\sqrt{5}}\rho\cos\theta \\ y = 2\rho\,\mathrm{sen}\,\theta \end{cases} \quad \begin{matrix} \rho\in[0,1] \\ \theta\in[0,2\pi] \end{matrix}$$

$$= \frac{4}{\sqrt{5}}\int_0^1 d\rho \int_0^{2\pi} \rho\left(4-4\rho^2\left(\frac{1}{5}\cos^2\theta+\mathrm{sen}^2\theta\right)\right)4\rho^2\left(\mathrm{sen}^2\theta-\frac{1}{5}\cos^2\theta\right)d\theta$$

$$= \frac{4}{\sqrt{5}}\int_0^1 d\rho \int_0^{2\pi}\left[16\rho^3\left(\mathrm{sen}^2\theta-\frac{1}{5}\cos^2\theta\right)+16\rho^5\left(\frac{1}{25}\cos^4\theta-\mathrm{sen}^4\theta\right)\right]d\theta$$

$$= \frac{64}{\sqrt{5}}\left[\frac{1}{4}\int_0^{2\pi}\left(\mathrm{sen}^2\theta-\frac{1}{5}\cos^2\theta\right)d\theta+\frac{1}{6}\int_0^{2\pi}\left(\frac{1}{25}\cos^4\theta-\mathrm{sen}^4\theta\right)d\theta\right]$$

$$= \frac{64}{\sqrt{5}}\left[\frac{\pi}{5}-\frac{3}{25}\pi\right] = \frac{128}{25\cdot\sqrt{5}}\pi$$