e poiché $\tilde{E}$ è normale rispetto l'asse t

$$\mathrm{vol}(\tilde{E}) = \int_0^{2\pi} d\theta \int_0^1 d\rho \int_0^{\frac{2}{1+\rho^2\cos^2\theta}} t^2 \rho \, dt =$$

$$= \int_0^{2\pi} d\theta \int_0^1 \frac{\rho}{3} \frac{8}{(1+\rho^2\cos^2\theta)^3} d\rho \overset{r=\rho^2}{=}$$

$$= \frac{4}{3} \int_0^{2\pi} d\theta \int_0^1 \frac{1}{(1+r\cos^2\theta)^3} dr = \quad \cos^2\theta \text{ è } \pi\text{-periodica}$$

$$= \frac{8}{3} \int_0^{\pi} d\theta \int_0^1 \frac{1}{(1+r\cos^2\theta)^3} dr = \quad s = 1+r\cos^2\theta,\ \theta \neq \frac{\pi}{2}$$

$$= \frac{4}{3} \int_0^{\pi} \frac{1}{\cos^2\theta} \left[-\frac{1}{s^2} \right]_1^{1+\cos^2\theta} d\theta = \frac{4}{3} \int_0^{\pi} \frac{1}{\cos^2\theta} \frac{\cos^4\theta + 2\cos^2\theta}{(1+\cos^2\theta)^2}$$

$\cos^2\theta$ simmetrica rispetto $\theta=\pi/2$

$$= \frac{8}{3} \int_0^{\pi/2} \frac{1}{\cos^2\theta} \frac{\cos^4\theta + 2\cos^2\theta}{(1+\cos^2\theta)^2} d\theta = \quad \cos^2\theta = \frac{1}{1+tg^2\theta},\ x = tg\theta$$

$$= \frac{8}{3} \int_0^{+\infty} \frac{2x^2+3}{(x^2+2)^2} dx = \frac{8}{3} \int_0^{+\infty} \left[\frac{1}{2} \frac{x^2}{(2+x^2)^2} + \frac{3}{2} \frac{1}{2+x^2} \right] dx$$

$$= \frac{4}{3} \int_0^{+\infty} \frac{x^2}{(x^2+2)^2} dx + 4 \int_0^{+\infty} \frac{1}{2+x^2} dx$$

$$= \frac{4}{3} \cdot \frac{\pi}{4} + 4 \frac{\pi}{\sqrt{2}} = \pi \left(\frac{1}{3} + 2\sqrt{2} \right)$$