quindi: $\forall x \in [0,+\infty)$

$$\lim_n f_n(x) = \lim_n g_n(x) = kx$$

Lo stesso ragionamento dà la convergenza uniforme su $[0,+\infty)$:

$$f_n(x) - kx = k\left(\frac{[nx]}{n} - x\right) + \frac{1}{n}\int_{[nx]}^{nx} |\mathrm{sen}(\pi t)|^{1/6}\,dt$$

$$\Rightarrow \sup_{[0,+\infty)} |f_n(x) - kx| \leq \frac{k+1}{n}$$

Un altro modo per dedurre la c.u. consiste nel cercare gli estremi relativi di $f_n(x) - kx$

$$\frac{d}{dx}(f_n(x) - kx) = |\mathrm{sen}(\pi n x)|^{1/6} - k \geq 0 \iff$$

$$|\mathrm{sen}(\pi n x)|^{\cancel{6}} \geq k^6 \iff \pi n x \in [\alpha, \pi - \alpha] + m\pi$$

$$(\text{con } \alpha = \mathrm{arcsen}\, k^6,\ m \in \mathbb{N}) \iff x \in \left[\frac{\alpha}{n\pi}, \frac{\pi-\alpha}{n\pi}\right] + \frac{m}{n}$$

da cui:

$$y_m = \frac{\alpha}{n\pi} + \frac{m}{n} \quad \text{min. rel.}$$

$$z_m = \frac{\pi - \alpha}{n\pi} + \frac{m}{n} \quad \text{max. rel.}$$

Infine:

$$|f_n(y_m) - ky_m| = \left| \cancel{\frac{1}{n}\int_0^m |\mathrm{sen}(\pi t)|^{1/6}\,dt} + \frac{1}{n}\int_m^{m+\frac{\alpha}{n\pi}} |\ldots|^{1/6}\,dt + \right.$$

$$\left. - k\frac{\alpha}{n\pi} - \cancel{k\frac{m}{n}} \right| \leq \frac{\alpha}{n\pi}\left(k + \frac{1}{n}\right)$$

analogamente per $|f_n(z_m) - kz_m|$.

Infine:

$$\sup_{[0,+\infty)} |f_n(x) - kx| = \sup_m \left(|f_n(y_m) - ky_m| \vee |f_n(z_m) - kz_m|\right)$$