ESERCIZIO 1

$f_n(-x) = -f_n(x)$, mi restringo allo studio in $[0,+\infty)$.

Idea: se $x \in \mathbb{N} \Rightarrow nx \in \mathbb{N} \Rightarrow$

$$f_n(x) = \frac{nx}{n} \int_0^1 |\mathrm{sen}(\pi t)|^{1/6} dt = kx$$

dato che $|\mathrm{sen}(\pi t)|^{1/6}$ è 1- periodica.

Se $x \in [0,+\infty) \setminus \mathbb{N}$, posto

$$g_n(x) = \frac{1}{n} \int_0^{[nx]} |\mathrm{sen}(\pi t)|^{1/6} dt$$

allora

$$f_n(x) = g_n(x) + \frac{1}{n} \int_{[nx]}^{nx} |\mathrm{sen}(\pi t)|^{1/6} dt$$

$$= \frac{[nx]}{n} \cdot k + \frac{1}{n} \int_{[nx]}^{nx} |\mathrm{sen}(\pi t)|^{1/6} dt$$

per concludere:

$$\frac{nx-1}{n} \leq \frac{[nx]}{n} \leq \frac{nx}{n} = x \Rightarrow \frac{[nx]}{n} \xrightarrow{n \to +\infty} x$$

$$\frac{1}{n} \int_{[nx]}^{nx} |\mathrm{sen}(\pi t)|^{1/6} dt \leq \frac{nx - [nx]}{n} \leq \frac{1}{n}$$