ESERCIZIO 6

Applicando il teorema della Divergenza al campo F:

$$\iiint_{\underbrace{\{r<\sqrt{x^2+y^2+z^2}<R\}}_{=B_{r,R}}} \operatorname{div} F \, dx\,dy\,dz = \int_{\partial B_{r,R}} F \cdot \nu_e \, d\sigma = I$$

d'altra parte:

$$\partial B_{r,R} = \underbrace{\left\{\sqrt{x^2+y^2+z^2}=r\right\}}_{= B_r} \cup \underbrace{\left\{\sqrt{x^2+y^2+z^2}=R\right\}}_{= B_R}$$

con
$$\nu_e = -\frac{(x\,y\,z)}{\sqrt{x^2+y^2+z^2}} \qquad (x\,y\,z) \in B_r$$
$$\nu_e = \frac{(x\,y\,z)}{\sqrt{x^2+y^2+z^2}} \qquad (x\,y\,z) \in B_R$$

da cui segue:

$$I = -\int_{B_r} f(r)\, d\sigma + \int_{B_R} f(R)\, d\sigma =$$
$$= 4\pi \left(R^2 f(R) - r^2 f(r)\right)$$