

$$\begin{aligned}
 &= \frac{1}{12} \int_{\pi/4}^{\pi/2} \cos \theta \sin^4 \theta \left[ \left(1 + \frac{1}{\sin^2 \theta}\right)^{3/2} - \left(1 + \frac{1}{4}\right)^{3/2} \right] d\theta \quad (2) \\
 &\stackrel{\substack{\uparrow \\ s = \sin \theta}}{=} \frac{1}{12} \int_{1/\sqrt{2}}^1 s^4 \left[ \frac{(1+s^2)^{3/2}}{s} - \frac{\sqrt{125}}{8} \right] ds = \\
 &\stackrel{\substack{\uparrow \\ n = s^2}}{=} \frac{1}{12} \left[ \int_{1/2}^1 \frac{n}{2} (1+n)^{3/2} dn - \frac{\sqrt{125}}{8 \cdot 5} \left(1 - \left(\frac{1}{\sqrt{2}}\right)^5\right) \right] = \\
 &\stackrel{\uparrow}{=} \frac{1}{12} \left[ \frac{1}{21} \left( \frac{2}{5} n (1+n)^{5/2} - \frac{2}{35} (1+n)^{7/2} \right) \right]_{1/2}^1 - \frac{\sqrt{5}}{8} \left(1 - \frac{1}{4\sqrt{2}}\right) \\
 &\text{integrando per parti} \\
 &= \frac{1}{12} \left[ \frac{4\sqrt{2}}{5} - \frac{2 \cdot 8 \cdot \sqrt{2}}{35} - \frac{1}{10} \left(\frac{3}{2}\right)^{5/2} + \frac{2}{35} \left(\frac{3}{2}\right)^{7/2} + \right. \\
 &\quad \left. - \frac{\sqrt{5}}{8} + \frac{\sqrt{5}}{32\sqrt{2}} \right]
 \end{aligned}$$

(3) Sia  $E$  il dominio cercato,  $\vec{e}$  è normale rispetto all'asse  $z$  e

$$V(E) = 2 \iint_D dx dy \int_0^{\sqrt{4+x^2+y^2}} dz = 2 \iint_D \sqrt{4+x^2+y^2} dx dy$$

dove

$$D = \left\{ (x,y) : \rho \in [0, 2|\cos \theta|^{1/2}]; \theta \in [0, 2\pi] \right\}$$