e quindi:

$$\frac{u(\gamma(t)) - u(\gamma(t_0))}{t-t_0} = \nabla u(\gamma(t_0)) \cdot \left(\frac{\gamma(t)-\gamma(t_0)}{t-t_0}\right) + \frac{o(\|\gamma(t)-\gamma(t_0)\|)}{t-t_0}$$

ma per $t \to t_0$ si ha:

$$\frac{\gamma(t)-\gamma(t_0)}{t-t_0} \longrightarrow \gamma'(t_0)$$

$$\frac{o(\|\gamma(t)-\gamma(t_0)\|)}{t-t_0} = \frac{o(\|\gamma(t)-\gamma(t_0)\|)}{\|\gamma(t)-\gamma(t_0)\|} \, \frac{\|\gamma(t)-\gamma(t_0)\|}{t-t_0}$$

e quindi:

$$\lim \frac{o(\|\gamma(t)-\gamma(t_0)\|)}{t-t_0} = 0$$

dato che

$\dfrac{\|\gamma(t)-\gamma(t_0)\|}{t-t_0}$ è limitato per $t \to t_0$

Quindi, si ha:

$$\lim_{t \to t_0} \frac{u(\gamma(t)) - u(\gamma(t_0))}{t-t_0} = \nabla u(\gamma(t_0)) \cdot \gamma'(t_0)$$

$$= \frac{\partial u}{\partial \tau}(\gamma(t_0))$$