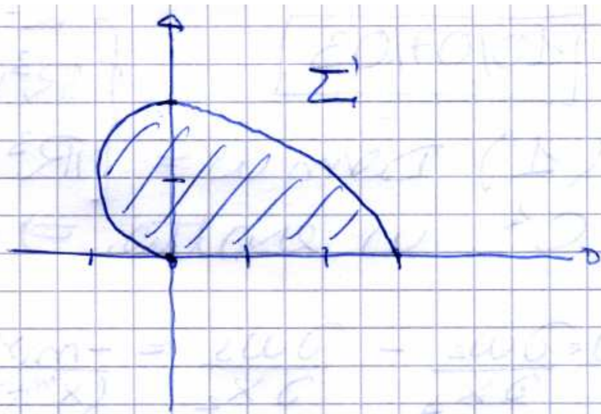
(3)




La curva in questione è l'arco di cardioide nel semipiano $y \geq 0$.
Allora

$$m(\Sigma') = \iint_{\Sigma'} (x^2+y^2)^{\frac12} \, dx\, dy = \int_0^\pi d\theta \int_0^{\rho(\theta)} \rho^2 \, d\rho =$$

$$= \frac13 \int_0^\pi 8 \underbrace{(1+\cos\theta)^3}_{1+c^3+3c^2+3c} d\theta = \frac83 \left(\pi + 3\int_0^\pi \cos^2\theta \, d\theta\right)$$

$$\frac83\pi + 8\left(\frac{\pi}{2}\right) = \frac{20}{3}\pi$$

$$m(\Sigma')\bar{x} = \iint_{\Sigma'} x(x^2+y^2)^{\frac12} \, dx\, dy = \int_0^\pi d\theta \int_0^{\rho(\theta)} \rho^3 \cos\theta \, d\rho$$

$$= \int_0^\pi \frac{16}{4}\cos\theta \underbrace{(1+\cos\theta)^4}_{1+c^4+4c^3+6c^2+6c} d\theta = \frac{16}{4}\int_0^\pi (c^5+4c^4+6c^3+4c^2+c)$$

$$= \int_0^\pi \frac{16}{4}(4c^4+4c^2)\, d\theta = 16\int_0^\pi \cos^4\theta \, d\theta + 16 \cdot \frac{\pi}{2} = 14\pi$$

per parti $\to = \frac14 \left[\cancel{\sin\theta\cos^3\theta}\right]_0^\pi + \frac34\int_0^\pi \cos^2\theta = \frac38\pi$

$\Rightarrow \bar{x} = 21/10$

$$m(\Sigma')\bar{y} = \iint_{\Sigma'} y(x^2+y^2)^{\frac12} \, dx\, dy = \int_0^\pi d\theta \int_0^{\rho(\theta)} \rho^3 \sin\theta \, d\rho =$$

$$= \int_0^\pi \frac{\cancel{16}^{\,4}}{\cancel{4}}(1+\cos\theta)^4 \sin\theta \, d\theta = \frac{\cancel{16}^{\,4}}{\cancel{4}}\int_{-1}^{1} (1+t)^4 \, dt = \left[\frac{(1+t)^5}{5}\right]_{-1}^{1} 16 = \frac{2^7}{5}$$