Se

$$F(x,y,z)=\left(\frac{1}{\sqrt{x^2+y^2+z^2}},\ x^2+2y-z^2,\ x+y+z\right)$$

$$\Delta F_2 = 2-2=0$$

$$\Delta F_3 = 0$$

$$\nabla F_1 = \frac{1}{(x^2+y^2+z^2)^{3/2}}\left(-\frac{1}{2}\right)2\,(x,y,z)$$

$$\frac{\partial^2 F_1}{\partial x^2} = \frac{-1}{(x^2+y^2+z^2)^3}\left((x^2+y^2+z^2)^{3/2}-x\,\frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}}\cdot 2x\right)$$

$$= \frac{-1}{(x^2+y^2+z^2)^{5/2}}\left(-2x^2+y^2+z^2\right)$$

poiché $\frac{\partial F_1}{\partial y}(x,y,z)=\frac{\partial F_1}{\partial x}(y,x,z)$ e $\frac{\partial F_1}{\partial z}(x,y,z)=$
$=\frac{\partial F_1}{\partial x}(z,y,x)$ si ha

$$\frac{\partial^2 F_1}{\partial y^2} = \frac{2y^2-x^2-z^2}{(x^2+y^2+z^2)^{5/2}}$$

$$\frac{\partial^2 F_1}{\partial z^2} = \frac{2z^2-x^2-y^2}{(x^2+y^2+z^2)^{5/2}}$$

da cui: $\Delta F_1 = 0$

e quindi: $$\iiint_\Omega (\ldots)\,dx\,dy\,dz = 0$$

(3) Una parametrizzazione dell'elica è data da

$$\begin{cases} x = \rho\cos\theta \\ y = \rho\,\mathrm{sen}\,\theta \\ z = \theta \end{cases}$$

$(\rho,\theta)\in[0,+\infty)\times\mathbb{R}$.