Da cui per determinare Σ basta imporre

$$0 \le (\rho\cos\theta)^2 + (\rho\,\mathrm{sen}\,\theta)^2 \le 1 - \frac{\theta^2}{4\pi^2} \iff$$

$$\rho \le \sqrt{1 - \frac{\theta^2}{4\pi^2}} \quad \text{e quindi } |\theta| \le 2\pi$$

Allora:

$$I = \int_\Sigma \sqrt{x^2+y^2}\, d\sigma = \int_{-2\pi}^{2\pi} d\theta \int_0^{\sqrt{1-\frac{\theta^2}{4\pi^2}}} \rho\, |\underline{X}_\rho \wedge \underline{X}_\theta|\, d\rho\, d\theta$$

ma

$$X_\rho \wedge X_\theta = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \cos\theta & \mathrm{sen}\,\theta & 0 \\ -\rho\,\mathrm{sen}\,\theta & \rho\cos\theta & 1 \end{vmatrix} =$$

$$= (\mathrm{sen}\,\theta; -\cos\theta; \rho)$$

$$\Rightarrow |X_\rho \wedge X_\theta| = \sqrt{\rho^2+1}$$

Allora

$$I = \int_{-2\pi}^{2\pi} d\theta \int_0^{\sqrt{1-\frac{\theta^2}{4\pi^2}}} \rho\sqrt{\rho^2+1}\, d\rho =$$

$$= \int_{-2\pi}^{2\pi} \frac{1}{2}\left[\frac{2}{3}(\rho^2+1)^{3/2}\right]_0^{\sqrt{1-\frac{\theta^2}{4\pi^2}}} d\theta =$$

$$= \frac{1}{3}\int_{-2\pi}^{2\pi} \left(\sqrt[3/2]{2 - \frac{\theta^2}{4\pi^2}} - 1\right) d\theta = \qquad t = \mathrm{sen}\, s$$

$$= -\frac{4}{3}\pi + \frac{2^{3/2}}{3}(8\pi^2)^{3/2} \int_{-\frac{2\pi}{(8\pi^2)^{3/2}}}^{\frac{2\pi}{(8\pi^2)^{3/2}}} (1-t^2)^{3/2}\, dt =$$