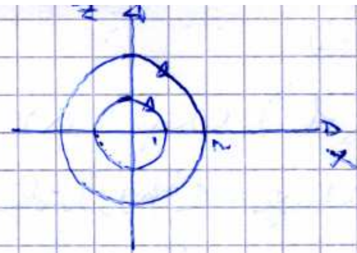
si ha $\int_{C_1^-} F \cdot t = 2$



da cui:

$$\int_{C_r^-} F \cdot t = \begin{cases} +2\pi(r^2-1)+2 & r \in [1,+\infty) \\ -(2\pi(1-r^2)-2) & r \in (0,1] \end{cases}$$
$$= 2\pi(r^2-1)+2$$

(4) $F \in C^2(\Omega, \mathbb{R}^3)$, $\Omega \subseteq \mathbb{R}^3$ aperto:

$$\nabla(\operatorname{div} F) = \nabla\left(\frac{\partial F_1}{\partial x_1} + \frac{\partial F_2}{\partial x_2} + \frac{\partial F_3}{\partial x_3}\right) =$$

$$= \left(\frac{\partial^2 F_1}{\partial x_1^2} + \frac{\partial^2 F_2}{\partial x_1 \partial x_2} + \frac{\partial^2 F_3}{\partial x_1 \partial x_3}, \frac{\partial^2 F_1}{\partial x_2 \partial x_1} + \frac{\partial^2 F_2}{\partial x_2^2} + \frac{\partial^2 F_3}{\partial x_2 \partial x_3}, \frac{\partial^2 F_1}{\partial x_3 \partial x_1} + \frac{\partial^2 F_2}{\partial x_3 \partial x_2} + \frac{\partial^2 F_3}{\partial x_3^2}\right)$$

$$\operatorname{rot} F = \left(\frac{\partial F_3}{\partial x_2} - \frac{\partial F_2}{\partial x_1}; \frac{\partial F_3}{\partial x_1} - \frac{\partial F_1}{\partial x_2}; \frac{\partial F_1}{\partial x_2} - \frac{\partial F_2}{\partial x_1}\right)$$

$$\operatorname{rot}(\operatorname{rot} F) = \left(\frac{\partial^2 F_3}{\partial x_3 \partial x_1} - \frac{\partial^2 F_1}{\partial^2 x_3} - \frac{\partial^2 F_1}{\partial^2 x_2} + \frac{\partial^2 F_2}{\partial x_2 \partial x_1}; \ldots\right)$$

da cui (almeno per la 1ª componente, gli altri conti sono analoghi) si ha

$$\nabla(\operatorname{div} F) - \operatorname{rot}(\operatorname{rot} F) = (\Delta F_1, \Delta F_2, \Delta F_3)$$

Quindi l'integrale che si vuole calcolare è:

$$\iiint_\Omega (\Delta F_1 \cdot x + \Delta F_2 \cdot y + \Delta F_3 \cdot z)\, dx\, dy\, dz$$