$$Cx\left(4\ln|x|+4+\cancel{\ln^2|x|}+2\ln|x|-\cancel{\ln^2|x|}\right)+$$
$$+Dx\left(2+\cancel{\ln|x|}+1-\cancel{\ln|x|}\right)=x\ln|x| \iff$$

$$\begin{cases}6C=1\\4C+3D=0\end{cases} \iff C=\frac{1}{6}\,;\,D=-\frac{2}{9}$$

Quindi:

$$y'=z=A|x|+B|x|^{-1/2}+\frac{1}{6}x\ln^2|x|-\frac{2}{9}x\ln|x| \quad (*)$$

da cui:

$$y(x)=A\int_0^x|t|dt+B\int_0^x|t|^{-1/2}dt+\frac{1}{6}\int_0^x t\ln^2|t|dt$$
$$-\frac{2}{9}\int_0^x t\ln|t|dt+C$$

e quindi:

$$y(x)=\frac{A}{2}x|x|+2B\,\mathrm{sgn}\,x\sqrt{|x|}+C+$$
$$\frac{1}{6}x^2\left(\frac{\ln^2|x|}{2}-\frac{\ln|x|}{2}+\frac{1}{4}\right)+\frac{2}{9}x^2\left(\frac{\ln|x|}{2}-\frac{1}{4}\right)$$
$$=\frac{A}{2}x|x|+2B\,\mathrm{sgn}\,x\sqrt{|x|}+x^2\left(\frac{\ln^2|x|}{12}+\right.$$
$$\left.+\frac{\ln|x|}{36}-\frac{1}{72}\right)+C$$

Poiché $\forall A,B,C\in\mathbb{R}$ $y$ può essere estesa con continuità in $x=0$ ponendo $y(0)=C$, $y$ risulta essere derivabile in tale punto sse $B=0$ (vedi $(*)$).