ESERCIZIO 4.

Sia $D \subseteq \mathbb{R}^3$ dato da

$$\{(x,y,z) \in \mathbb{R}^3 : x \geq 0, y \geq 0, z \geq 0, 4x^2+y^2+z^2 \leq 4\}$$

allora: $\partial D = \Sigma \cup \Sigma_x \cup \Sigma_y \cup \Sigma_z$

dove:

$$\Sigma_z = \{(x,y,z) : z = 0 \quad 4x^2+y^2 \leq 4\}$$

$$\Sigma_y = \{ = : y = 0 \quad 4x^2+z^2 \leq 4\}$$

$$\Sigma_x = \{ = : x = 0 \quad y^2+z^2 \leq 4\}$$

Adesso per il Teorema della Divergenza:

$$\iiint_D \operatorname{div} \vec{F}\, dx\,dy\,dz = \int_{\partial D} \vec{F} \cdot \nu_{ext}\, d\sigma =$$

$$= \int_{\Sigma} \vec{F} \cdot \nu_{ext}\, d\sigma + \int_{\Sigma_x} \vec{F} \cdot (-1,0,0)\, d\sigma + \int_{\Sigma_y} \vec{F} \cdot (0,-1,0)\, d\sigma + \int_{\Sigma_z} \vec{F} \cdot (0,0,-1)\, d\sigma$$

($-F_1$, $-F_2$, $-F_3$)

con $\vec{F} = (F_1, F_2, F_3)$.

si ha:

$$\operatorname{div} \vec{F} = x + z - x + z + x - 3 + 4z - x - x$$

$$= 4z$$