$x = \mathrm{tg}\, 1$ è massimo relativo (nota che $1 < \sqrt{2}$ ③
$\Rightarrow \mathrm{tg}\, 1 < \mathrm{tg}\sqrt{2}$, quindi $x = \mathrm{tg}\, 1$ è il punto che si
era trovato mediante il teorema di Weierstrass).

Se $x \in (0, +\infty) \setminus \{\mathrm{tg}\sqrt{2}\}$:

$$f''(x) = -\frac{8}{3}\Big\{ 2f(x)f'(x)\frac{\mathrm{atn}\, x}{1+x^2}(\mathrm{atn}^2 x - 1)^{1/3} +$$

$$+ \frac{f^2(x)}{(1+x^2)^2}\Big[(1+x^2)\Big(\frac{(\mathrm{atn}^2 x - 1)^{1/3}}{1+x^2} + \frac{\mathrm{atn}^2 x}{1+x^2}\cdot\frac{2}{3}(\mathrm{atn}^2 x - 1)^{-2/3}\Big)$$

$$- 2x\, \mathrm{atn}\, x\, (\mathrm{atn}^2 x - 1)^{1/3}\Big]\Big\}$$

$$= -\frac{8}{3}\frac{f^2(x)}{(1+x^2)^2}\Big\{-2f(x)\frac{8}{3}\mathrm{atn}^2 x\,(\mathrm{atn}^2 x - 1)^{2/3} + (\mathrm{atn}^2 x - 1)^{1/3}$$

$$+ \frac{2}{3}\mathrm{atn}^2 x\,(\mathrm{atn}^2 x - 1)^{-2/3} - 2x\, \mathrm{atn}\, x\,(\mathrm{atn}^2 x - 1)^{1/3}\Big\}$$

$$= +\frac{8}{3}\frac{f^2(x)}{(1+x^2)^2}(\mathrm{atn}^2 x - 1)^{-2/3}\Big\{+\frac{16}{3}f(x)\,\mathrm{atn}^2 x\,(\mathrm{atn}^2 x - 1)^{4/3} +$$

$$-(\mathrm{atn}^2 x - 1 + \frac{2}{3}\mathrm{atn}^2 x) + 2x\,\mathrm{atn}\, x\,(\mathrm{atn}^2 x - 1)\Big\}$$

da cui si deduce:

$$\lim_{x\to 0^+} f'' = -\infty = \lim_{x\to(\mathrm{tg}\sqrt{2})^-} f'' = -\lim_{x\to(\mathrm{tg}\sqrt{2})^+} f''$$

Infine si noti che se $x \geq \mathrm{tg}\sqrt{2}$ allora

$$\{\ldots\} \underset{f(x)>0}{\geq} (\mathrm{atn}^2 x - 1)(2x\,\mathrm{atn}\, x - 1) - \frac{2}{3}\mathrm{atn}^2 x$$

$$\underset{x\cdot\mathrm{atn}\, x \geq \sqrt{2}}{\geq} \mathrm{atn}^2 x\Big(2\sqrt{2} - \frac{5}{3}\Big) - (2\sqrt{2} - 1) > 0$$

$\Rightarrow f$ convessa su $(\mathrm{tg}\sqrt{2}, +\infty)$