ESERCIZIO 6: Determinare il polinomio P di ⑦ grado minimo per cui

$$\lim_{x\to\pi/2}\left(\ln\left(\frac{2}{\pi}x\right)-2e^{\left(x-\frac{\pi}{2}\right)^2}-P(x)\right)\mathrm{tg}^3 x=1$$

Poiché $\cos x=\cos\left(x-\frac{\pi}{2}+\frac{\pi}{2}\right)=\operatorname{sen}\left(\frac{\pi}{2}-x\right)$ e $\operatorname{sen} t=t+o(t^2)$ $t\to 0$, si cerca P t.c.

$$\frac{\ln\left(\frac{2}{\pi}x\right)-2e^{\left(x-\frac{\pi}{2}\right)^2}-P(x)}{-\left(x-\frac{\pi}{2}\right)^3}\xrightarrow{x\to\frac{\pi}{2}}1$$

Se $t=x-\frac{\pi}{2}$ e $Q(t)=P\left(t+\frac{\pi}{2}\right)$ il limite si riscrive come:

$$\ln\left(\frac{2}{\pi}t+1\right)-2e^{t^2}-Q(t)=-t^3+o(t^3)\iff$$

$$\frac{2}{\pi}t-\frac{1}{2}\left(\frac{2}{\pi}t\right)^2+\frac{1}{3}\left(\frac{2}{\pi}t\right)^3+o(t^3)-2-2t^2-t^4+o(t^4)-Q(t)$$

$$=-2+\frac{2}{\pi}t-\left(\frac{2}{\pi^2}+2\right)t^2+\frac{8}{3\pi^3}t^3-Q(t)+o(t^3)=-t^3+o(t^3)$$

da cui

$$Q(t)=-2+\frac{2}{\pi}t-\left(\frac{2}{\pi}+2\right)t^2+\left(\frac{8}{3\pi^3}+1\right)t^3$$

e

$$P(x)=-2+\frac{2}{\pi}\left(x-\frac{\pi}{2}\right)-\left(\frac{2}{\pi}+2\right)\left(x-\frac{\pi}{2}\right)^2$$

$$+\left(\frac{8}{3\pi^3}+1\right)\left(x-\frac{\pi}{2}\right)^3$$