Integrando si ottiene: (3)

$$F(x) = \begin{cases} 2\sqrt{2}\left(1-\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)\right) & x\in\left[-\frac{\pi}{2},\pi\right] \\ 2\sqrt{2}\left(\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)-1\right) & x\in\left[-\pi,-\frac{\pi}{2}\right] \end{cases}$$

<u>2° modo:</u> Si usano le formule parametriche, i.e.

$t=\operatorname{tg}\frac{x}{2}$, $\operatorname{sen} x = \frac{2t}{1+t^2}$

$$\int\sqrt{1+\operatorname{sen}x}\,dx = \int \frac{2|1+t|}{(1+t^2)^{3/2}}\,dt$$

D'altra parte:

$$\int \frac{1+t}{(1+t^2)^{3/2}}\,dt \overset{\pm t^2}{=} \int\left[\frac{1}{(1+t^2)^{1/2}}-\frac{t^2}{(1+t^2)^{3/2}}+\frac{t}{(1+t^2)^{3/2}}\right]dt$$

$$\overset{\text{per parti}}{=} \operatorname{settsh} t - \left[t\int\frac{t}{(1+t^2)^{3/2}} - \int\left(\int\frac{t}{(1+t^2)^{3/2}}dt\right)\right]+$$

$$-(1+t^2)^{-1/2}$$

$$= \operatorname{settsh} t + (t-1)(1+t^2)^{-1/2} - \int\frac{1}{\sqrt{1+t^2}}\,dt = \frac{t-1}{\sqrt{1+t^2}} + \text{cost}.$$

e quindi

$$F(x) = \int_{-\frac{\pi}{2}}^{x}\sqrt{1+\operatorname{sen}s}\,ds = \begin{cases} \frac{2(\operatorname{tg}(x/2)-1)}{(1+\operatorname{tg}^2(x/2))^{1/2}}+2^{3/2}, & x\in(-\pi,\pi),\ \operatorname{tg}\frac{x}{2}\geq -1 \\ -\frac{2(\operatorname{tg}(x/2)-1)}{(1+\operatorname{tg}^2(x/2))^{1/2}}-2^{3/2}, & x\in(-\pi,\pi),\ \operatorname{tg}\frac{x}{2}\leq -1 \end{cases}$$

dato che $\operatorname{tg}\frac{x}{2}\geq -1 \quad x\in(-\pi,\pi) \iff x\in\left(-\frac{\pi}{2},\pi\right)$.

Si conclude quindi: