$$I = \underbrace{\int_{-\pi}^{-\pi/2} x\sqrt{1+\operatorname{sen} x}\, dx}_{I_1} + \underbrace{\int_{-\pi/2}^{\pi/2} x\sqrt{1+\operatorname{sen} x}\, dx}_{I_2} + \underbrace{\int_{\pi/2}^{\pi} x\sqrt{1+\operatorname{sen} x}\, dx}_{I_3} \quad 5$$

e quindi

$$I_1 \underset{\substack{t=\operatorname{sen} x \\ x=-\operatorname{arcsen} t-\pi}}{=} + \int_0^{-1} (\operatorname{arcsen} t + \pi)\, \overset{\frac{1}{\sqrt{1-t}}}{\cancel{\frac{\sqrt{1+t}}{\sqrt{1-t^2}}}}\, dt$$

$$I_2 \underset{\substack{t=\operatorname{sen} x \\ x=\operatorname{arcsen} t}}{=} \int_{-1}^{1} \frac{\operatorname{arcsen} t}{\sqrt{1-t}}\, dt$$

$$I_3 \underset{\substack{t=\operatorname{sen} x \\ x=\pi-\operatorname{arcsen} t}}{=} - \int_{1}^{0} \frac{(\operatorname{arcsen} t + \pi)}{\sqrt{1-t}}\, dt$$

da cui:

$$I = -\pi \int_{-1}^{0} \frac{1\, dt}{\sqrt{1-t}} + \pi \int_0^1 \frac{1}{\sqrt{1-t}}\, dt$$

$$= +2\pi \left(+(1-t)^{+1/2}\Big|_{-1}^{0} - (1-t)^{+1/2}\Big|_{0}^{1} \right)$$

$$= 2\pi\left(2-\sqrt{2}\right)$$