

ESERCIZIO 1: Sia

(1)

$$a_n = \ln((n^{2/5} + 5)^{1/3} - (n^{2/5} - 5)^{1/3}) + \frac{4}{15} \ln n$$

poiché

$$\begin{aligned} (n^{2/5} + 5)^{1/3} - (n^{2/5} - 5)^{1/3} &= \frac{10}{(n^{2/5} + 5)^{2/3} + (n^{2/5} + 5)^{1/3}(n^{2/5} - 5)^{1/3} + (n^{2/5} - 5)^{2/3}} \\ &= \frac{10}{n^{4/15} (3 + o(1))} \end{aligned}$$

si ottiene

$$a_n = \ln \frac{10}{3} - \frac{4}{15} \ln n - \ln(1 + o(1)) + \frac{4}{15} \ln n$$

da cui

$$\lim_{n \rightarrow +\infty} a_n = \ln \frac{10}{3}$$