ESERCIZIO 2 Sia

$$f(x) = \left(\sqrt{x^2-2x+2}-x+1\right)\ln\left(\sqrt{x^2-2x+2}-x+1\right)$$

$$t = \sqrt{x^2-2x+2}-x \quad \left(\Longleftrightarrow \quad x = \frac{t^2-2}{2(1+t)}\right)$$

si ottiene

$$\int f(x)\,dx = \int \cancel{(t+1)}\ln(t+1)\,\frac{t^2+2t+2}{2(1+t)^{\cancel{2}\,1}}\,dt$$

$$= \frac{1}{2}\int \ln(t+1)\left(t+1+\frac{1}{1+t}\right)dt \overset{s=t+1}{=}$$

$$= \frac{1}{2}\int \left(s+\frac{1}{s}\right)\ln s\,ds = \frac{1}{2}\left(\frac{s^2}{2}\ln s - \frac{s^2}{4} + \frac{\ln^2 s}{2}\right)+c$$

Quindi

$$\int f(x)\,dx = \frac{1}{4}\Big[\left(\sqrt{x^2-2x+2}-x+1\right)\left(\ln\left(\sqrt{x^2-2x+2}-x\right)-\frac{1}{2}\right)$$

$$+ \ln^2\left(\sqrt{x^2-2x+1}-x+1\right)\Big]+c$$