$\boxed{\text{ESERCIZIO 4}}$

Il numeratore $n(x)$ si riscrive come:

$$n(x) = \underbrace{\ln(x^2)\left(e^{-\operatorname{atn} x} - 1\right)}_{} + \ln(\cos x + \operatorname{sen} x - x)$$

$$= \underbrace{\frac{e^{-\operatorname{atn} x} - 1}{-\operatorname{atn} x}}_{\longrightarrow 1} \cdot \underbrace{\frac{(-\operatorname{atn} x)}{-x}}_{\longrightarrow 1} \underbrace{(-x \ln(x^2))}_{\longrightarrow 0} +$$

$$\underbrace{\frac{\ln(\overbrace{\cos x}^{\longrightarrow 1} + \overbrace{\operatorname{sen} x - x}^{\longrightarrow 0})}{-1+\cos x + \operatorname{sen} x - x}}_{\longrightarrow 1} \cdot \left(\underbrace{\frac{\cos x - 1}{x^2}}_{\longrightarrow -1/2} + \underbrace{\frac{\operatorname{sen} x - x}{x^2}}_{\longrightarrow 0} \right) \cdot x^2$$

e quindi $n(x) \xrightarrow{x \to 0^+} 0$.

Per il denominatore invece si ha

$$d(x) = 1 - (1-19x)^{\ln x} = 1 - e^{\ln x \ln(1-19x)}$$

e poiché:

$$\ln x \cdot \ln(1-19x) = \underbrace{\frac{\ln(1-19x)}{-19x}}_{\longrightarrow 1} \cdot \underbrace{(-19x \ln x)}_{\longrightarrow 0}$$

anche $d(x) \xrightarrow{x \to 0^+} 0$.

Inoltre:

$$d(x) = \underbrace{\frac{1 - e^{\ln x \ln(1-19x)}}{-\ln x \cdot \ln(1-19x)}}_{\longrightarrow 1} \underbrace{\frac{\ln(1-19x)}{-19x}}_{\longrightarrow 1} 19 x \ln x.$$

Quindi:

$$\frac{n(x)}{d(x)} = (1+o(1)) \cdot \left(-\frac{2}{19}\right) + (1+o(1))\left(-\frac{1}{2}+o(1)\right) \frac{x}{19 \ln x}$$

da cui:

$$\lim_{x \to 0^+} \frac{n(x)}{d(x)} = -\frac{2}{19}$$