$\boxed{\text{ESERCIZIO 1:}}$ Sia

$$a_n = \frac{((n+2)!)^{n-1} - ((n-1)!)^{n+1}}{(n^2 n!)^{n-1}}$$

si deve provare che

$$\lim_{n\to+\infty} a_n = e^3.$$

Si noti che:

$$\frac{((n+2)!)^{n-1}}{(n^2 n!)^{n-1}} = \Bigg(\frac{\overbrace{(n+2)(n+1)}^{=n^2+3n+1}}{n^2}\Bigg)^{n-1}$$

$$= \left(1+\frac{3n+1}{n^2}\right)^{n-1} = \left[\left(1+\frac{3n+1}{n^2}\right)^{\frac{n^2}{3n+1}}\right]^{\frac{(3n+1)(n-1)}{n^2}}$$

da cui ricordando che:

$$a_n \longrightarrow +\infty \implies \left(1+\frac{1}{a_n}\right)^{a_n} \longrightarrow e$$

si ha

$$\frac{((n+2)!)^{n-1}}{(n^2 n!)^{n-1}} \xrightarrow{n\to+\infty} e^3. \qquad (1)$$

Inoltre, si ha:

$$\frac{((n-1)!)^{n+1}}{(n^2 n!)^{n-1}} = \frac{((n-1)!)^2}{n^{2(n-1)}} \frac{((n-1)!)^{n-1}}{(n!)^{n-1}}$$

$$= \left(\frac{(n-1)!}{n^{n-1}}\right)^2 \cdot \frac{1}{n^{n-1}}$$

e poiché: $\frac{n!}{n^n} \xrightarrow{n\to+\infty} 0$ si ha:

$$\frac{((n-1)!)^{n+1}}{(n^2 n!)^{n-1}} \xrightarrow{n\to+\infty} 0. \qquad (2)$$

Concludendo, da (1) e (2) segue: $a_n \xrightarrow{n\to+\infty} e^3$.