$$I_2 = \frac{1}{2}\int \sqrt{r+1}\,dr = \frac{1}{3}\cdot(r+1)^{3/2} + c$$

$$\underset{r=s^2}{=} \frac{1}{3}(s^2+1)^{3/2} + c \underset{s=\frac{2t+1}{3}=\frac{2\,\mathrm{sen}\,x+1}{3}}{=} \frac{1}{3}\left(\left(\frac{2\,\mathrm{sen}\,x+1}{3}\right)^2+1\right)^{3/2} + c$$

I_3 si ottiene per sostituzione ponendo $s = \mathrm{sh}\,y$:

$$I_3 = \int \mathrm{ch}^2 y\,dy \underset{\mathrm{ch}^2 y = \frac{\mathrm{ch}(2y)+1}{2}}{=} \frac{1}{2}\int(\mathrm{ch}(2y)+1)\,dy$$

$$= \frac{1}{4}\mathrm{sh}(2y) + \frac{y}{2} + c = \frac{1}{2}\left(s\sqrt{1+s^2} + \mathrm{settsh}\,s\right) + c$$

$$= \frac{1}{2}\left(s\sqrt{s^2+1} + \ln\left(s+\sqrt{s^2+1}\right)\right) + c \underset{s=\frac{2t+1}{3}=\frac{2\,\mathrm{sen}\,x+1}{3}}{=}$$

$$= \frac{1}{2}\left(\frac{2\,\mathrm{sen}\,x+1}{3}\cdot\sqrt{\left(\frac{2\,\mathrm{sen}\,x+1}{3}\right)^2+1} + \right.$$

$$\left.\ln\left(\frac{2\,\mathrm{sen}\,x+1}{3} + \sqrt{\left(\frac{2\,\mathrm{sen}\,x+1}{3}\right)^2+1}\right)\right) + c$$

Concludendo, poiché:

$$\int f(x)\,dx = -\frac{3}{2}\cos(2x) + I_1 = -\frac{3}{2}\cos(2x) + \frac{27}{2}I_2 - \frac{9}{2}I_3$$

si ottiene:

$$\int f(x)\,dx = -\frac{3}{2}\cos(2x) + \frac{1}{6}\big(\overbrace{4\,\mathrm{sen}^2 x + 4\,\mathrm{sen}\,x + 10}^{g(x)}\big)^{3/2} +$$

$$-\frac{1}{4}(2\,\mathrm{sen}\,x+1)\left(g(x)\right)^{1/2} +$$

$$-\frac{9}{4}\left(\ln\left(2\,\mathrm{sen}\,x+1+(g(x))^{1/2}\right) - \ln 3\right) + c$$