$\boxed{\text{ESERCIZIO 5}}$ Studiamo la convergenza di (7)

$$\int_0^{\pi/2} \underbrace{\frac{\ln\left(\frac{2}{\pi}x\right)}{\ln(\operatorname{sen} x)} \sqrt{\frac{\pi}{2x}-1}}_{f(x)}\, dx$$

Si noti che $\mathrm{Dom} f \equiv \left(0,\frac{\pi}{2}\right)$, determiniamo il comportamento di f in $x=0$, $x=\frac{\pi}{2}$.

Poiché $\operatorname{sen} x = x + o(x^2)$ $x \to 0$ si ha

$$f(x) = \frac{\ln\frac{2}{\pi} + \ln x}{\ln(x+o(x^2))} \sqrt{\frac{\pi}{2x}-1}$$

$$= \frac{\ln\frac{2}{\pi} + \ln x}{\ln(1+o(x))+\ln x} \sqrt{\frac{\pi}{2x}-1} \qquad x \longrightarrow 0^+$$

e quindi $\lim\limits_{x\to 0^+} f = \lim\limits_{x\to 0^+} \sqrt{\frac{\pi}{2x}-1} = +\infty$.

In $x=\frac{\pi}{2}$ si ha: $\operatorname{sen} x = \operatorname{sen}\left(\frac{\pi}{2}+\left(x-\frac{\pi}{2}\right)\right) = \cos\left(x-\frac{\pi}{2}\right)$ da cui

$$\operatorname{sen} x = 1 - \frac{1}{2}\left(x-\frac{\pi}{2}\right)^2 + o\left(\left(x-\frac{\pi}{2}\right)^3\right)$$

e quindi:

$$\ln(\operatorname{sen} x) = \ln\left(1-\frac{1}{2}\left(x-\frac{\pi}{2}\right)^2 + o\left(\left(x-\frac{\pi}{2}\right)^3\right)\right)$$

$\ln(1+t)=t+o(t)$

$$= -\frac{1}{2}\left(x-\frac{\pi}{2}\right)^2 + o\left(\left(x-\frac{\pi}{2}\right)^2\right)$$

Inoltre

$$\ln\left(\frac{2}{\pi}x\right) = \ln\left(1+\frac{2}{\pi}\left(x-\frac{\pi}{2}\right)\right)$$

$$= \frac{2}{\pi}\left(x-\frac{\pi}{2}\right) + o\left(x-\frac{\pi}{2}\right)$$

e quindi segue