ESERCIZIO 1: Fissato $k \in \mathbb{N}$ sia

$$a_n(k) = \left(\frac{(n+1)^{n+k+1}}{(n+k+1)^{n+1}} - n^k \right) \operatorname{tg} \frac{1}{n^k}$$

proviamo che

$$\lim_{n \to +\infty} a_n(k) = e^{-k} - 1 .$$

Infatti:

$$a_n(k) = \left((n+1)^k \cdot \frac{(n+1)^{n+1}}{(n+k+1)^{n+1}} - n^k \right) \operatorname{tg} \frac{1}{n^k}$$

$$= \Bigg(\underbrace{\left(1+\frac{1}{n}\right)^k}_{\longrightarrow 1} \cdot \frac{1}{\underbrace{\left(1+\frac{k}{n+1}\right)^{n+1}}_{\longrightarrow e^k}} - 1 \Bigg) \underbrace{n^k \operatorname{tg} \frac{1}{n^k}}_{\longrightarrow 1}$$

da cui la tesi.