$\boxed{\text{ESERCIZIO 2}}$ Calcolare

$$\int_{e^{-2}}^{e^{2}} \frac{1}{3x+x|\ln^2 x-1|}\,dx =: I$$

Usando la sostituzione $t=\ln x$ si ottiene

$$I=\int_{-2}^{2}\frac{1}{3+|t^2-1|}\,dt = 2\int_{0}^{2}\frac{1}{3+|t^2-1|}\,dt$$

$f(t)=\frac{1}{3+|t^2-1|}$ pari

$$=2\int_0^1 \frac{1}{4-t^2}\,dt + 2\int_1^2 \frac{1}{t^2+2}\,dt = I_1+I_2$$

Poiché
$$\frac{1}{4-t^2}=\frac{A}{2-t}+\frac{B}{2+t} \iff \begin{cases} 2(A+B)=1 \\ A-B=0\end{cases} \iff \begin{cases} A=1/4 \\ A=B\end{cases}$$
si ha

$$I_1=\frac{1}{2}\left[\ln\left|\frac{2+t}{2-t}\right|\right]_0^1=\frac{1}{2}\cdot\ln 3$$

Inoltre:

$s=\frac{t}{\sqrt{2}}$

$$I_2=\frac{\overset{1}{\cancel{2}}}{\cancel{2}}\int_1^2\frac{1}{\left(\frac{t}{\sqrt{2}}\right)^2+1}\,dt=\sqrt{2}\int_{\frac{1}{\sqrt{2}}}^{\sqrt{2}}\frac{1}{1+s^2}\,ds$$

$$=\sqrt{2}\,\left[\operatorname{atn} s\right]_{\frac{1}{\sqrt{2}}}^{\sqrt{2}}=\sqrt{2}\left(\operatorname{atn}\sqrt{2}-\operatorname{atn}\frac{1}{\sqrt{2}}\right)$$

$$=2\sqrt{2}\,\operatorname{atn}\sqrt{2}-\frac{\pi}{\sqrt{2}}$$

Concludendo:

$$I=\frac{1}{2}\ln 3+2\sqrt{2}\,\operatorname{atn}\sqrt{2}-\frac{\pi}{\sqrt{2}}$$