(4) Sia
$$f(x) = \frac{x^{3/2}\ln x}{(1+\operatorname{atn} x)^{(x^x-1)}-1}$$

allora $\operatorname{Dom} f = (0,+\infty)\setminus\{1\}$, quindi

$$I = \int_0^{+\infty} f(x)\,dx = \underbrace{\int_0^{1/2} f(x)\,dx}_{I_1} + \underbrace{\int_{1/2}^{1} f(x)\,dx}_{I_2} + \underbrace{\int_1^{3/2} f(x)\,dx}_{I_3} + \underbrace{\int_{3/2}^{+\infty} f(x)\,dx}_{I_4}$$

visto che:

$$f(x) > 0 \iff \begin{cases} \ln x > 0 & (<0) \\ (1+\operatorname{atn} x)^{(x^x-1)} > 1 & (<1) \end{cases}$$

essendo $x>0 \Rightarrow 1+\operatorname{atn} x > 1$, e poiché

$$1 < x^x = e^{x\ln x} \iff x\ln x > 0 \iff x > 1$$

si ha: $f(x) > 0 \quad \forall x \in \operatorname{Dom} f$

Quindi, I_i, $1 \le i \le 4$, sono ben definiti e si possono applicare i Thm di Cfr e Cfr Asintotico.

$\underline{\underline{I_1}}$:
$$(1+\operatorname{atn} x)^{(x^x-1)} = e^{(x^x-1)\ln(1+\operatorname{atn} x)} =$$
$$1 + (x^x-1)\ln(1+\operatorname{atn} x) + o((x^x-1)x) \overset{x^x = e^{x\ln x}}{=}$$
$$1 + x^2\ln x + o(x^2 \ln x)$$

da cui $f(x) = \dfrac{1}{x^{1/2}+o(x^{1/2})}$ e quindi se $g(x) = \dfrac{1}{x^{1/2}}$

$$\frac{f(x)}{g(x)} \xrightarrow{x\to 0^+} 1 \Rightarrow I_1 \text{ converge}$$