b) La proposizione è vera per $n=0$:

$$\sum_{k=0}^{0} k\binom{n}{k} = 0 = 0\cdot 2^{-1}$$

Supponiamola vera per n e dimostriamo che allora segue:

$$\sum_{k=0}^{n+1} k\binom{n+1}{k} = (n+1)\cdot 2^{n}$$

Infatti:

$$\sum_{k=0}^{n+1} k\binom{n+1}{k} = \sum_{k=1}^{n+1} k\binom{n+1}{k} = \sum_{k=1}^{n} k\binom{n+1}{k} + (n+1)$$

$$= \sum_{k=1}^{n} k\binom{n}{k} + \sum_{k=1}^{n} k\binom{n}{k-1} + (n+1) =$$

$$= \sum_{k=0}^{n} k\binom{n}{k} + \sum_{k=0}^{n-1} (k+1)\binom{n}{k} + (n+1) =$$

$$= n\,2^{n-1} + \sum_{k=0}^{n-1} k\binom{n}{k} + \sum_{k=0}^{n-1}\binom{n}{k} + (n+1) =$$

$$= n\cdot 2^{n-1} + \sum_{k=0}^{n} k\binom{n}{k} + \sum_{k=0}^{n}\binom{n}{k} = n\cdot 2^{n-1} + n\,2^{n-1} + 2^{n}$$

$$= n\cdot 2^{n} + 2^{n} = (n+1)\cdot 2^{n}$$