Si ha:
$$\frac{e^{-\frac{1}{x^{2\alpha}}}}{e^{-\frac{1}{x^2}}} = e^{-\frac{1}{x^{2\alpha}}+\frac{1}{x^2}}$$

e quindi:
$$\lim_{x\to 0}\frac{e^{-\frac{1}{x^{2\alpha}}}}{e^{-\frac{1}{x^2}}} \overset{t=\frac{1}{x^2}}{=} \lim_{t\to+\infty} e^{-t^\alpha+t} =$$

$$= \begin{cases} 0 & \alpha>1 \\ 1 & \alpha=1 \\ +\infty & \alpha<1 \end{cases}$$

Si noti che per $\alpha=1$ si ha $\ln\alpha=0$ e quindi:
$$\lim_{x\to 0} f(x) = 1 \neq f(0).$$

Se $\boxed{\alpha>1:}$ $\ln\left(e^{-\frac{1}{x^{2\alpha}}}+e^{-\frac{1}{x^2}}\right) = \ln\left(e^{-\frac{1}{x^2}}\left(1+e^{-\frac{1}{x^{2\alpha}}+\frac{1}{x^2}}\right)\right)$

$$= -\frac{1}{x^2}+\ln\Big(1+\underbrace{e^{\frac{1}{x^2}-\frac{1}{x^{2\alpha}}}}_{\to 0 \quad x\to 0}\Big)$$

e quindi:
$$\lim_{x\to 0} x^2\ln\left(e^{-\frac{1}{x^{2\alpha}}}+e^{-\frac{1}{x^2}}\right) = -1$$

da cui: $\lim_{x\to 0} f(x) = 1+\ln\alpha = 0 \iff \alpha = e > 1.$

Se $\boxed{\alpha<1}$: $\ln\left(e^{-\frac{1}{x^{2\alpha}}}+e^{-\frac{1}{x^2}}\right) = \ldots = -\frac{1}{x^{2\alpha}}+\ln\Big(1+\underbrace{e^{\frac{1}{x^{2\alpha}}-\frac{1}{x^2}}}_{\xrightarrow{x\to 0} 0}\Big)$

da cui:
$$\lim_{x\to 0} x^2\ln\left(e^{-\frac{1}{x^{2\alpha}}}+e^{-\frac{1}{x^2}}\right) = 0$$

e quindi: $\lim_{x\to 0} f(x) = 1 \neq f(0).$

In conclusione, f risulta continua su tutto $\mathbb{R}$ se e solo se $\alpha=e$.