$$I_2 = \int (-6)(-1+\text{ch}^2 t)\,\text{ch}^2 t\,dt = \int 6(\text{ch}^2 t - \text{ch}^4 t)\,dt$$

A questo punto ricordando la def. di $\text{ch}\,t$ si possono calcolare le potenze e ridursi a integrare un polinomio in e^t.
Altrimenti si può procedere anche così:

$$\int \text{ch}^4 t\,dt = \text{sh}\,t\,\text{ch}^3 t - \int 3\,\text{ch}^2 t\,\text{sh}^2 t\,dt =$$

$$= \text{sh}\,t\,\text{ch}^3 t - 3\int \text{ch}^4 t\,dt + 3\int \text{ch}^2 t\,dt$$

da cui:

$$\int \text{ch}^4 t\,dt = \frac{1}{4}\text{sh}\,t\,\text{ch}^3 t + \frac{3}{4}\int \text{ch}^2 t\,dt$$

infine:

$$\int \text{ch}^2 t\,dt = \text{sh}\,t\,\text{ch}\,t - \int \text{sh}^2 t\,dt$$

da cui:

$$\int \text{ch}^2 t\,dt = \frac{1}{2}\text{sh}\,t\,\text{ch}\,t + \frac{t}{2} + k$$

quindi:

$$I_2 = \frac{3}{4}\text{sh}\,t\,\text{ch}\,t + \frac{3}{4}t - \frac{3}{2}\text{sh}\,t\,\text{ch}^3 t$$

Con la sostituzione $c = \text{ch}\,t$ si ottiene

$$I_3 = \frac{\cancel{12}^{\,4}}{\cancel{3}}\,\text{ch}^3 t$$

Concludendo, ricordando che $\text{ch}^2 t = x^2+4x+5$

$$\int f(x)\,dx = \frac{(x^2+4x+5)^{5/2}}{5} + (x^2+4x+5)^{3/2}\left(\frac{11}{3} - \frac{3}{2}(x+2)\right)$$
$$- \frac{13}{4}\left((x+2)(x^2+4x+5)^{1/2} + \text{settsh}(x+2)\right)$$
$$+ \frac{x^5}{5} + \frac{x^4}{2} + k$$