08/09/03

① $$\lim_{x\to 0} \frac{tg^4(\pi e^x)}{\cos(\operatorname{sen} x)+Ch(\operatorname{sh} x)-2e^{x^4}}$$

F.I. 0/0, ma

$$Tg(\pi e^x) = Tg((e^x-1)\pi+\pi) = +Tg(\pi(e^x-1))$$

$$= \frac{Tg(\pi(e^x-1))}{\pi(e^x-1)}\pi(e^x-1)$$

$$\cos(\operatorname{sen}x) = 1-\frac{(\operatorname{sen}x)^2}{2}+\cancel{o(\operatorname{sen}^3 x)}\frac{(\operatorname{sen}x)^4}{4!}+$$

$$+o(\operatorname{sen}^5 x) = 1-\frac{1}{2}\left(x-\frac{x^3}{6}+o(x^4)\right)^2+$$

$$+\frac{(x+o(x^2))^4}{4!}+o(x^5)$$

$$= 1-\frac{x^2}{2}+\frac{x^4}{6}+\frac{x^4}{4!}+o(x^5)$$

$$= 1-\frac{x^2}{2}+\frac{5}{24}x^4+o(x^5)$$

$$Ch(\operatorname{sh}x) = 1+\frac{(\operatorname{sh}x)^2}{2}+\frac{(\operatorname{sh}x)^4}{4!}+o(\operatorname{sh}^5 x) =$$

$$= 1+\frac{1}{2}\left(x+\frac{x^3}{6}+o(x^4)\right)^2+\frac{1}{4!}(x+o(x))^4+o(x^5)$$

$$= 1+\frac{x^2}{2}+\frac{x^4}{6}+\frac{x^4}{24}+o(x^5)$$

$$2e^{x^4} = 2+2x^4+o(x^4)$$

$$\cos(\operatorname{sen}x)+Ch(\operatorname{sh}x)-2e^{x^4} = \frac{5}{12}x^4-2x^4+o(x^4)$$

$$= -\frac{19}{12}x^4+o(x^4)$$

Quindi il limite in questione è uguale al limite

$$\lim_{x\to 0} -12\frac{\pi^4}{19}\left(\frac{e^x-1}{x}\right)^4 = -\frac{12}{19}\pi^4$$