② $\int \frac{x+\operatorname{sen}x}{1+\cos x}\,dx = I$

1° modo: $I = \int \frac{x}{1+\cos x}\,dx + (-\ln(1+\cos x))$

$$\int \frac{x}{1+\cos x}\,dx = x\int \frac{1}{1+\cos x}\,dx - \int\left(\int \frac{dx}{1+\cos x}\right)dx$$

$$\int \frac{1}{1+\cos x}\,dx = \frac{1}{2}\int \frac{2}{1+\cos x}\,dx \overset{t=\frac{x}{2}}{=} \int \frac{dt}{\cos^2 t} = \operatorname{tg}\left(\frac{x}{2}\right)$$

da cui:

$$\int \frac{x}{1+\cos x}\,dx = x\operatorname{tg}\frac{x}{2} - \int \operatorname{tg}\frac{x}{2}\,dx =$$
$$= x\operatorname{tg}\frac{x}{2} + 2\ln\left|\cos\frac{x}{2}\right| + c$$

quindi:

$$I = x\operatorname{tg}\frac{x}{2} + 2\ln\left|\cos\frac{x}{2}\right| - \ln(1+\cos x) + c$$
$$= x\operatorname{tg}\frac{x}{2} + c$$

2° modo:

$$I = (x+\operatorname{sen}x)\int \frac{1}{1+\cos x}\,dx - \int (1+\cos x)\left(\int \frac{dx}{1+\cos x}\right)dx$$
$$= (x+\operatorname{sen}x)\operatorname{tg}\frac{x}{2} - \int (1+\cos x)\operatorname{tg}\frac{x}{2}\,dx$$
$$= (x+\operatorname{sen}x)\operatorname{tg}\frac{x}{2} + 2\ln\left|\cos\frac{x}{2}\right| +$$
$$- \int \frac{2\cos^2\left(\frac{x}{2}\right)-1}{\cos\left(\frac{x}{2}\right)}\operatorname{sen}\frac{x}{2}\,dx$$
$$= (x+\operatorname{sen}x)\operatorname{tg}\frac{x}{2} + 2\ln\left|\cos\frac{x}{2}\right| + 2\int \frac{2c^2-1}{c}\,dc$$