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Approximating singular ODEs using finite difference and collocation codes

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Abstract

Boundary value problems in ODEs with singularities arise in numerous mathematical models describing real-life phenomena in natural sciences and engineering. This motivates vivid research activities aiming to characterize the analytical properties of singular problems, to investigate convergence of the standard numerical methods when they are applied to simulate differential equation with singularities, and to provide software for their efficient numerical treatment. There are two well-known, high order numerical methods which we focus on in this paper, the finite difference schemes and the collocation methods. Those methods proved to be dependable and highly accurate in the context of regular differential equations, so the question arises how do they preform for singular problems. While, there is a strong evidence for the collocation schemes to be a robust method to solve singular systems in a stable and efficient way, finite difference schemes are still considered less suitable for this problem class.
In this paper, we shall compare the performance of the code `HOFiD_bvp` [1] based on the high order finite difference schemes and `bvpsuite2.0` [2] based on polynomial collocation, when the codes are applied to singular problems in ODEs. We are fully aware of the difficulties in a code comparison, so here, we will try to only diagnose the potential improvements, we could address in the next update of the codes.

References

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