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A Magnus based exponential integrator for stochastic oscillators

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Abstract

In this talk, we propose a numerical method for solving second order scalar stochastic differential equations of the form

$$\ddot{x} = -\omega^2(t)x + f(t, x(t)) + \sigma(t)\xi(t) \tag{1}$$

where $f : [0, T] \times \mathbb{R}$ and $\xi(t)$ is a white noise process satisfying $\mathbb{E}|\xi(t)\xi(t')| = \delta(t - t')$. Equation (1) describes a class of nonlinear oscillators with non-constant frequency. The proposed scheme extends the idea of Magnus integrators to the stochastic case and its convergence properties are here rigorously analyzed. Selected numerical experiments on relevant stochastic oscillators confirm the effectiveness and the competitive behaviour of the method, with respect to standard integrators.

References

1. S. Blanes, F. Casas, J.A. Oteo and J. Ros, The Magnus expansion and some of its applications, Phys. Rep. 470 (2009), 151–238.
2. R. D'Ambrosio, H. de la Cruz, C. Scalone, A Magnus-based integrator for Brownian parametric semi-linear oscillators, Appl. Math. Comp. 472 (2024), 128610.
3. M. Gitterman, The noisy oscillator, The first hundred years, from Einstein until now, World Scientific (2005).

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