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Quantifying the discretization error in the numerical integration of evolution equations via Bayesian isotonic regression

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Abstract

In the numerical analysis of differential equations, understanding the discretization error is of significant importance. Error bounds, often dependent on parameters such as time step size, are typically derived through theoretical analysis. However, there is a growing demand for more practical methods to *quantify* the discretization error, especially in the context of data assimilation and computational uncertainty quantification.

The authors have recently proposed methods for error quantification based on the empirical observation that discretization error tends to increase almost monotonically as time integration progresses [1,2,3]. However, these studies have focused on point estimation, limiting their direct applicability to Bayesian frameworks, which are crucial in data assimilation.

In this presentation, we introduce a new method for discretization error quantification using Bayesian isotonic regression. The challenge lies in the choice of an appropriate prior that reflects the monotonicity assumption. We address this issue by employing a horseshoe prior [4]. Additionally, we discuss efficient sampling algorithms within the proposed framework, enhancing its practical utility.

References

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