The conjugate symplectic scheme of the Gauss Runge-Kutta method of order 4

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Abstract

It is well know that the classical couple of symplectic/conjugate-symplectic schemes is given by Midpoint (the Gauss RK method of order 2) and the Trapezoidal Rule. They can be considered as composition of the Implicit and Explicit Euler methods in direct and reverse order. To the best of our knowledge, an analogous relation for higher order Gauss methods is not known. Moreover in [1] only the trapezoidal rule is reported as non symplectic but conjugate symplectic method up to order ∞. All the other methods are conjugate symplectic up to a finite order. Following the theory in [2] and [3], where a new couple of order 4 symplectic and conjugate symplectic methods is analyzed, we presents here a new method which is conjugate-symplectic to the order 4 Gauss Runge-Kutta method. We start by deriving the classical fourth order Gauss method considering a multiderivative extension of the mid-point scheme and approximating the derivatives with the collocation polynomial using the zeros of the 2nd order shifted Legendre polynomial. This allows us to derive a high order dense output [4] and, starting from the class of multiderivative extension of the trapezoidal scheme, the associated conjugate symplectic method.

References

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