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Spectral Collocation Solutions to some Nonlinear BVPs on Infinite Domains. Applications to Physico-Chemical Hydrodynamics

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Abstract

We are concerned with some accurate spectral Chebyshev collocation (ChC) solutions, to some nonlinear and singular BVPs on real line. Thus, some draining or coating fluid-flow problems can be described by a third-order ODE of the form

$$u_{xxx} = f(u)\, u_x + g(x, u, \delta)\,,\ -\infty < x < \infty \tag{1}$$

along with the boundary condition $u \to 1$ as $x \to -\infty$, so modelling a layer of fluid that is asymptotically uniform behind the draining front and $u \to \delta$ as $x \to \infty$ where $0 < \delta \ll 1$, so describing the draining over an already-wet surface. The actual form of the functions f and g vary according to the physical context, and we take into account here some simple rational function of u of the form δ^s/u^n, $s \geq 0,\ n > 1$. Unfortunately, the solution $u(x, \delta)$ has some unacceptable features when u is small, associated ultimately with the impossibility of moving a contact line $u = 0$ over a nonslip dry wall.

In order to overcome all the difficulties we will implement ChC in the form given by Chebfun. In this way we will get correct information on the accuracy of the computed solution. Actually, the solution u to (1) is the steady state of an unsteady process described by the initial/BVP

$$u_t = \left(u_{xxx} - f(u)\, u_x + g(x, u, \delta)\right)_x, -\infty < x < \infty,\ t > 0, \tag{2}$$

with similar boundary conditions as above and an appropriate initial condition. ChC along with a FD scheme for stiff problems have succeeded in solving (2).

This approach confirms our opinion that nonlinear and singular BVPs formulated on the real line can be more accurately solved by spectral collocation, than resorting to the classical shooting.

References

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