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A VEM approach for the solution of the 2D linear elastic wave equation via scalar potentials

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Abstract

Soft tissues and other nearly incompressible media pose a challenge for simulating elastic wave propagation, due to the slow propagation of shear waves compared to pressure waves. To overcome this challenge, a classical Helmholtz-Hodge decomposition is used to split the displacement field into scalar pressure (P-) and shear (S-) waves, allowing for separate treatment of the two dynamics and the construction of discretization spaces suited for each type of wave.

This presentation focuses on the simulation of 2D soft scattering elastic wave propagation in isotropic homogeneous media, using the scalar potential decomposition in the time-harmonic regime. For problems defined in bounded domains, a Virtual Element Method (VEM) with varying mesh sizes and degrees of accuracy is proposed to approximate the two scalar potentials. For unbounded domains, a boundary element method is coupled with the VEM.

The proposed approach performs better than standard methods that directly use the vector formulation, as it allows for tracking the different wave numbers associated with P- and S-speeds of propagation. This makes it possible to use a high-order method for the approximation of waves with higher wave numbers. We establish the stability of our method and present a convergence error estimate in the L^2-norm for the displacement field. Notably, our estimate separates the contributions to the error associated with the P- and S- waves. We provide numerical results to demonstrate the effectiveness of the proposed approach.

References

S. Falletta, M. Ferrari, L. Scuderi, *A virtual element method for the solution of 2D time-harmonic elastic wave equations via scalar potentials*, Journal of Computational and Applied Mathematics, 441 (2024), 115625.

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