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Modified equations and backward error analysis for stochastic optimization algorithms

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Abstract

Stochastic optimizations algorithms, such as *stochastic gradient methods* and its variants, are effective tools for numerically solving optimization problems since they allow to dramatically reduce computational costs. For example, it turns out that when the dimension of the problem becomes higher and higher, a widely employed algorithm, among the scientific community, is the so-called *stochastic coordinate descent method* (SCD) [3]. In this talk, we provide first attempts of exporting the principle of the *weak backward error analysis* technique [1, 4] to analyze qualitative properties of such algorithm. The theoretical analysis will rely on the study of the so-called *weak modified equations* [4], associated to such method. Specifically, a mean-square stability analysis for such stochastic differential equations will be presented, allowing to gain more insights on the qualitative convergent character of the aforementioned algorithm towards the unique minimizer of the object function. The theoretical details of this study has been presented in [2].

References

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