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Optimally Truncated WKB Approximation For The 1D Stationary Schrödinger Equation In The Semi-classical Limit

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Abstract

In this talk we discuss the numerical solution of the 1D stationary Schrödinger equation $\varepsilon^2\varphi'' + a(x)\varphi = 0$ for $0 < \varepsilon \ll 1$ in the oscillatory regime, i.e. for $a(x) > 0$. In order to approximate the solution, we build upon the well-known WKB-ansatz, which relies on an asymptotic expansion w.r.t. the small parameter ε. To compute the WKB approximation, we apply highly accurate spectral methods (already used in [1]) for the calculation of the functions occuring in the asymptotic series. Moreover, by establishing estimates for these functions, we are able to analyze the error of the WKB approximation explicitly. This allows us to prove that the error is exponentially small w.r.t. ε if the underlying asymptotic series is truncated optimally, i.e., in the sense of minimizing the resulting error. The corresponding optimal truncation order is found to be proportional to $1/\varepsilon$.

References

[1] A. Arnold, C. Klein, B. Ujvari, WKB-method for the 1D Schrödinger equation in the semi-classical limit: enhanced phase treatment, *BIT Numerical Mathematics* **62**, 1–22 (2022).

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