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Weighted least squares collocation methods

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Abstract

For the numerical solution of the initial value problem (without loss of generality we assume the problem scalar)

$$y' = f(t,y), \qquad y(t_0) = y_0 \in \mathbb{R}, \qquad t \in [t_0, T],$$

we consider the overdetermined collocation method

$$\begin{cases} u(t_0) = y_0, \\ u'(t_0 + c_i h) = f(t_0 + c_i h, u(t_0 + c_i h)), \quad i = 1, \dots, k, \end{cases}$$

where $h > 0$ is the stepsize, $0 \le c_1 < c_2 < \cdots < c_k \le 1$ are the collocation abscissae, $u(t)$ is a polynomial of degree at most s and $k \ge s$.

We propose a weighted least squares approach to derive a numerical solution. Denoting by Π_s the vector space of polynomials of degree at most s, for a given distribution of (positive) weights $\{w_i\}_{i=1,\dots,k}$ satisfying the normalization condition $\sum_{i=1}^k w_i = 1$, the approximating polynomial $u^* \in \Pi_s$ is defined by

$$u^* = \underset{u \in \Pi_s, u(t_0) = y_0}{\arg\min} \sum_{i=1}^{k} w_i \left(u'(t_0 + c_i h) - f(u(t_0 + c_i h)) \right)^2. \tag{1}$$

The discrete problem requires the evaluation of the Jacobian of the vector field which, however, appears in a $O(h)$ term, h being the stepsize. We show that, by neglecting this infinitesimal term, the resulting scheme becomes a low-rank Runge–Kutta method. Among the possible choices of the weights distribution, we analyze the one based on the quadrature formula underlying the collocation conditions.

References

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