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Hypocoercivity for linear ODEs and strong stability for Runge-Kutta methods

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Abstract

In this talk we present two topics that sound rather distinct initially, and then reveal their connection.

Linear autonomous ODE-systems $\dot{x}(t) = -Cx(t)$ with a matrix $C \in \mathbb{C}^{n\times n}$ that is not coercive, are called hypocoercive if they still exhibit uniform exponential decay towards the steady state $x \equiv 0$. For semi-dissipative matrices $-C$ (i.e., with $C + C^H \geq 0$), $\|x(t)\|$ is monotonically decreasing. For such systems, the *index of hypocoercivity* is defined via a coercivity-type estimate for the Hermitian/skew-Hermitian parts of C, and it describes the interplay between these two parts. In [AAC] it was recently proven that this index, m_{HC}, characterizes the polynomial short-time decay of the propagator norm:

$$\|e^{-Ct}\| = 1 - c\,t^{2m_{HC}+1} + \mathcal{O}(t^{2m_{HC}+2}) \quad \text{for } t \to 0\,,$$

with some constant $c > 0$ and $\|\cdot\|$ denoting the spectral norm.

Strong stability is a property of time integration schemes for ODEs that preserve temporal monotonicity of solutions in arbitrary (inner product) norms. We show that explicit Runge–Kutta schemes of order $p \in 4\mathbb{N}$ with $s = p$ stages are not strongly stable, a case left open in [SS]. Furthermore, for explicit Runge–Kutta methods of order $p \in \mathbb{N}$ and $s > p$ stages, sufficient conditions on the stability function are derived to ensure strong stability of the Runge-Kutta scheme. In several situations, the hypocoercivity index needs to be small enough (more precisely, $2m_{HC} + 1 \leq p$) to guarantee strong stability [AAJ].

References

[AAC] F. Achleitner, A. Arnold, E.A. Carlen: The hypocoercivity index for the short time behavior of linear time-invariant ODE systems. Journal of Differential Equations 371 (2023), 83–115.
[AAJ] F. Achleitner, A. Arnold, A. Jüngel: Necessary and sufficient conditions for strong stability of explicit Runge–Kutta methods. preprint 2023.
[SS] Z. Sun, C.-W. Shu: Strong stability of explicit Runge–Kutta time discretizations. SIAM J. Numer. Anal. 57 (2019), 1158–1182.

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