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Numerical solution of linear fractional integro-differential equations

Mikk Vikerpuur*, Arvet Pedas

Abstract

We consider a class of fractional weakly singular integro-differential equations

$$(D^{\alpha_p}_{Cap}y)(t) + \sum_{i=0}^{p-1} d_i(t)(D^{\alpha_i}_{Cap}y)(t) + \sum_{i=0}^{q} \int_0^t (t-s)^{-\kappa_i} K_i(t,s)(D^{\theta_i}_{Cap}y)(s)ds = f(t), \quad 0 \le t \le b, \quad (1)$$

subject to the conditions

$$\sum_{j=0}^{n_0} \beta_{ij0}\, y^{(j)}(0) + \sum_{k=1}^{l}\sum_{j=0}^{n_1} \beta_{ijk}\, y^{(j)}(b_k) + \beta_i \int_0^{\bar{b}_i} y(s)ds = \gamma_i\,, \quad i = 0,\dots,n-1. \quad (2)$$

Here D^{δ}_{Cap} is the Caputo differential operator of order $\delta > 0$ and $n := \lceil \alpha_p \rceil$ is the smallest integer greater or equal to the highest fractional order α_p. We assume that: $0 \le \alpha_0 < \alpha_1 < \dots < \alpha_p \le n$, $0 \le \theta_j < \alpha_p$, $0 \le \kappa_j < 1$, $j = 0,\dots,q$, with $p \in \{1,2,\dots\}$ and $q \in \{0,1,\dots\}$, the given functions d_i ($i = 0,\dots,p-1$), K_j ($j = 0,\dots,q$) and f are continuous on their respective domains, $0 \le n_0, n_1 < n$, $b_k \in (0,b]$ ($k = 1,\dots,l$) and $\bar{b}_i \in (0,b]$ ($i = 0,\dots,n-1$).

Following [1], we reformulate (1)–(2) as a Volterra integral equation of the second kind with respect to the fractional derivative $D^{\alpha_p}_{Cap}y$. We then regularize the solution by a suitable smoothing transformation and solve the transformed integral equation by a piecewise polynomial collocation method on a mildly graded or uniform grid. We show the convergence of the proposed algorithm and present global superconvergence results for a class of specific collocation parameters. Finally, we complement the theoretical results with some numerical examples.

References

[1] A. Pedas, M. Vikerpuur, *Spline collocation for multi-term fractional integro-differential equations with weakly singular kernels*, Fractal Fract. 5 (2021), 90.

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* University of Tartu. Narva 18, Tartu, Estonia; email: mikk.vikerpuur@ut.ee