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A family of fourth order energy-preserving integrators

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Abstract

We consider a system of ordinary differential equations possessing a first integral. Such a system is written as

$$\frac{\mathrm{d}}{\mathrm{d}t}y = S(y)\nabla H(y), \quad y(0) = y_0 \in \mathbb{R}^m, \tag{1}$$

where $S(y) \in \mathbb{R}^{m\times m}$ is a skew-symmetric matrix and $H(y)$ is a scalar function, called energy hereafter. We assume that both $S(y)$ and $H(y)$ are suitably regular. Along with the solution to the system (1), the energy $H(y)$ is constant. We are concerned with a numerical integrator preserving $H(y)$, i.e. $H(y_n) = H(y_0)$ for $n = 1, 2, \dots$, where y_n denotes an approximate solution to $y(t_n)$.

Several approaches to constructing energy-preserving integrators have been proposed [1,2,3]. Among them, Cohen and Hairer [3] consider an extension of the AVF method. This method can be arbitrary high-order while the integrator is uniquely determined for a specific order. In other words, the method does not embrace free parameters. However, integrators with free parameters might be advantageous in terms of, for example, accuracy and efficiency, and further might have the potential to preserve other structures of the system.

In this talk, we propose a family of fourth-order energy-preserving integrators with some free parameters. The construction can be viewed as an extension of the AVF method. More precisely, we consider a partitioned continuous-stage Runge–Kutta (PCSRK) method with three stages and characterize the order conditions and energy-preserving conditions for (1) in terms of the PCSRK coefficients. With some choices of remaining free parameters, we can build, for example, a parallelizable integrator.

References

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