ICNAAM 2022
14th Symposium on Recent Trends in the Numerical Solution of Differential Equations

Mean-square dissipative stochastic Runge-Kutta methods

Stefano Di Giovacchino

Abstract

In this talk, we deal with stochastic Runge-Kutta methods [4] applied to mean-square dissipative stochastic differential equations [1, 3]. It is well-known that a one-sided Lipschitz drift and a globally Lipschitz diffusion make the correspondent stochastic system dissipative in the mean-square sense. Then, our aim is to analyze the behaviour of stochastic Runge-Kutta methods applied to such equations to capture their eventual ability to inherit the mean-square contractivity also along their numerical dynamics. The investigation will allow us to discover that a dissipation, in mean-square sense, is achieved, under suitable algebraic constraints on the coefficients of the method and the presence of a spurious term (that, in principle, may affect any eventual contractive character of the method [2]) is also visible. However, we show that this spurious term vanishes for increasing sizes of the time window of the integration and for smaller values of the stepsize Δt, making the determined conditions on the coefficients of the method sufficient for observing a good long-term behaviour of the related stochastic Runge-Kutta method, in terms of mean-square contractivity. Finally, selected numerical examples are provided to confirm the theoretical results. This is a joint work with Raffaele D'Ambrosio (University of L'Aquila).

References

[1] R. D'Ambrosio, S. Di Giovacchino, *Mean-square contractivity of stochastic ϑ-methods*, Commun. Nonlinear Sci. Numer. Simul., vol. 96, no. 105671 (2021).

[2] R. D'Ambrosio, S. Di Giovacchino, *Nonlinear stability issues for stochastic Runge-Kutta methods*, Commun. Nonlinear Sci. Numer. Simul., vol. 94, no. 105549 (2021).

[3] D.J. Higham, P.E. Kloeden, *Numerical methods for nonlinear stochastic differential equations with jumps*, Numer. Math., vol. 101, pp. 101–119 (2005).

[4] A. Rössler, *Runge-Kutta methods for the strong approximation of solutions of stochastic differential equations*, SIAM J. Numer. Anal., vol. 48, pp. 922–952 (2010).

---

* DISIM, University of L'Aquila, Via Vetoio - Loc. Coppito, 67100 L'Aquila (AQ), Italy, stefano.digiovacchino@graduate.univaq.it