13-th Symposium on Recent Trends in the Numerical Solution of Differential Equations
ICNAAM 2020
18-th International Conference of Numerical Analysis and Applied Mathematics
Rhodes (Greece), September 17-23, 2020

High order methods for multi-term fractional integro-differential equations with weakly singular kernels

Arvet Pedas, Mikk Vikerpuur

Institute of Mathematics and Statistics, University of Tartu; Narva street 18, 51009, Tartu, Estonia
tel: +372 737 5863, email: mikk.vikerpuur@ut.ee

We consider a class of multi-term fractional weakly singular integro-differential equations with non-local boundary conditions:

$$(D^{\alpha_p}_{Cap}y)(t) + \sum_{i=1}^{p-1} h_i(t)(D^{\alpha_i}_{Cap}y)(t) + h_0(t)y(t) + \int_0^t (t-s)^{-\kappa} K(t,s)y(s)ds = f(t),$$

$$\gamma_0 y(0) + \gamma_1 y(b_1) + \gamma_2 \int_0^{b_2} y(s)ds = \gamma.$$

Here $0 \le t \le b$, $b > 0$, $b_1, b_2 \in (0, b]$, $\gamma_0, \gamma_1, \gamma_2, \gamma \in \mathbb{R} := (-\infty, \infty), p \in \mathbb{N} := \{1, 2, \dots\}$ and $D^{\alpha_i}_{Cap}$ are the Caputo differential operators of orders $\alpha_i \in (0, 1), i = 1, \dots, p$. More precisely, we assume that $0 < \alpha_1 < \alpha_2 < \dots < \alpha_p < 1$, $\kappa \in [0, 1), f \in C[0, b]$, $h_i \in C[0, b]$ $(i = 0, \dots, p-1)$, $K \in C(\Delta_b)$, $\Delta_b = \{(t, s) : 0 \le s \le t \le b\}$ and $\gamma_0 + \gamma_1 + \gamma_2 b_2 \neq 0$. Note that this class also involves settings for fractional initial value problems ($\gamma_0 \neq 0$, $\gamma_1 = \gamma_2 = 0$) and fractional two-point boundary value problems ($\gamma_0 \neq 0, \gamma_1 \neq 0$, $\gamma_2 = 0$). By using an integral equation reformulation, we first study the existence, uniqueness and smoothness of the exact solution. On the basis of this information and spline collocation techniques, the numerical solution of the problem is discussed. The convergence behaviour of the proposed algorithms is established and global error estimates are derived. A numerical illustration is also given.