13th Symposium on Recent Trends in the Numerical Solution of Differential Equations
ICNAAM 2020
18th International Conference of Numerical Analysis and Applied Mathematics
Rhodes (Greece), September 17–23, 2020.

Numerical conservation issues for stochastic Hamiltonian problems

Raffaele D'Ambrosio[1], Giuseppe Giordano[2], Beatrice Paternoster[2]

[1] *DISIM, University of L'Aquila, Via Vetoio, Loc. Coppito, L'Aquila - 67100, Italy.*
e-mail: *raffaele.dambrosio@univaq.it*
[2] *Department of Mathematics, University of Salerno, Via Giovanni Paolo II 132, Fisciano - 84084, Italy.*
e-mail: *gigiordano@unisa.it*, *beapat@unisa.it*; phone: +39 081 5293213

Stochastic differential equations (SDEs) are excellent models useful in a large number of applications, since they provide a key tool for a mesoscopic approach to describe the effects of external environments to a physical model. The irreversible character of a stochastic dynamics destroys the idea of isolate systems, since the particles are repeatedly influenced by small unpredictable perturbations of the external environment. In order to combine the laws of evolution for average macroscopic observables with the microscopic dynamics of the constituent particles of a statistical system, it is necessary that the microscopic dynamics have a highly chaotic character and the overall system can be decomposed into almost independent microscopic subsystems.

Specifically, stochastic Hamiltonian problems are the most suitable candidates to conciliate the Hamiltonian nature of classical mechanics, which is closely related to the canonical character of the evolution equations, with the non-differentiable nature of the Wiener process, which describes the continuous innovative character of stochastic effects.

We focus on the study of stochastic Hamiltonian problem driven by additive noise. Stochastic Runge-Kutta methods obtained as stochastic perturbation of symplectic Runge-Kutta methods exhibit a remarkable error growth as the parameter ϵ of the diffusive part increases. Through a perturbative theory, we investigate the reason of this behaviour, due to the presence of a secular term $\epsilon\sqrt{t}$ destroying the overall conservation accuracy. The theoretical expectations are also confirmed by selected numerical experiments.

References

[1] A. Bazzani, Hamiltonian systems and Stochastic processes, Lecture Notes, University of Bologna (2018).

[2] P.M. Burrage, K. Burrage, Structure-preserving Runge-Kutta methods for stochastic Hamiltonian equations with additive noise, Numer. Algorithms 65, 519–532 (2014).

[3] C. Chen, D. Cohen, R. D'Ambrosio, A. Lang, Drift-preserving numerical integrators for stochastic Hamiltonian systems, Adv. Comput. Math 46, 27 (2020).