A semi-implicit high-order space-time scheme on staggered meshes for the 2D incompressible Navier-Stokes equations

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A new high order accurate semi-implicit staggered space-time Discontinuous Galerkin method on unstructured grids of two-dimensional domains, for the simulation of viscous incompressible flows is presented. Following the ideas in [1] and [4], a *semi-implicit* discretization is employed, which combines the simplicity of explicit methods for nonlinear hyperbolic PDE with the stability and efficiency of implicit time discretizations. The designed scheme is of the *Arbitrary Lagrangian Eulerian* type, which is suitable to work on fixed as well as on moving meshes that are allowed to move with an almost arbitrary mesh velocity. In our space-time formulation, by expressing the numerical solution in terms of piecewise space-time polynomials, an arbitrary high order of accuracy in time is achieved through a simple and efficient *Picard's method*. This approach is inspired by the local space-time Galerkin predictor method proposed for the high order time discretization of P_N-P_M schemes in [2] and [3]. For the *staggered* dual mesh, the basis functions consist in the union of continuous piecewise polynomials on the two subtriangles within the quadrilaterals: this allows the construction of a quadrature-free scheme, resulting in a very efficient algorithm, not only from a computer memory point of view, but also concerning the required CPU time. Several numerical examples confirm that the proposed method outperform existing ones.

References

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