Spectral Collocation Based on Quasi-Classical Orthogonal Polynomials. Applications in Solving Nonlinear Singular BVPs.

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It is well established that spectral collocation methods based on classical orthogonal polynomials, in spite of their high order accuracy, use bad conditioned differentiation matrices, i.e., fully populated, rather non-normal and badly conditioned with respect to inversion.

The aim of this essay is to try to find other orthogonal polynomials, with respect to more sophisticated measures, which could generate better differentiation matrices and consequently more accurate collocation methods. We are mainly interested in solving boundary value problems on unbounded intervals. Thus, we analyse orthogonal polynomials with respect to the generalized Laguerre measure and the logistic measure ([2]).

The capabilities of some non-standard orthogonal polynomials in obtaining accurate quadrature formulas has been documented in the above quoted monograph. Unfortunately, few attempts have been made to use them in collocation methods. We want to fill this gap as much as possible

The benchmark problem we take into account is the so-called *density profile equation* which is a genuinely nonlinear one defined on the half line with singularities at both ends (see for instance [5] and [1] for analytical results). The spectral collocation methods considered cast the attached boundary value problem into a nonlinear system of algebraic equations. The combined effect of singularity and nonlinearity increases when a physical parameter approach the unity. We elaborate on the numerical methods to solve the nonlinear system analysing the outputs of some optimization algorithms such as first order optimality, elapsed CPU time, number of iterations and function counts etc.

The best and more reliable results are obtained by *sinc collocation* (see additionally our contributions [3] and [4]). The search for better collocation methods based on non standard orthogonal polynomials is still an open issue. The first attempts are rather less encouraging.

References

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