Diagonally Implicit Functional Continuous Methods

Alexey S. Eremin

Abstract

In the talk we consider Retarded Functional Differential Equations (RFDEs)

$$\dot{u}(t) = f(t, u_t)$$

where $u : [a-r, b) \to \mathbb{R}^d$ and u_t is the function in $\mathcal{C}^d$ given by $u_t(\theta) = u(t+\theta)$, $\theta \in [-r, 0]$.

Such problems cannot be solved with continuous extensions of explicit Runge–Kutta methods for ordinary differential equations — they become fully implicit since the solution must be known in the whole interval up to the current time point, i.e. within the step not yet calculated (situation known as overlapping).

Explicit Functional Continuous Runge–Kutta methods (FCRKs), suggested in early 1970s and published in details in 2005 by S. Maset, L. Torelli and R. Vermiglio, are specially constructed for direct implementation to RFDEs. They remain explicit for any kind of right-hand side f of the equation.

Better stability properties for stiff problems are provided with implicit Runge–Kutta methods among which diagonally implicit methods (DIRKs) are much faster and easier to implement than fully implicit. However the same problems with overlapping as for explicit methods arise.

On base of order conditions for FCRKs developed earlier we construct Functional Continuous DIRKs that are L-stability when applied to ODEs (in this first work on implicit FCRKs we don't consider other stability properties, those specific for DDE too). In order to reduce the number of necessary stages we use the first explicit stage. We construct L-stable methods of orders two, three and four (the latter is $L(\alpha)$ stable.