An $\mathcal{H}$-matrix based Direct Solver for the Boundary Element Method in 3D Elastodynamics

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The Boundary Element Method (BEM) has emerged as an efficient alternative to all the classical domain methods to handle seismic wave propagation [5]. The BEM is based on a boundary integral formulation, which requires the discretization of the only domain boundaries, leading to a drastic reduction of the total number of Degrees of Freedom (DoF) of the problem. A major drawback of classical BEM is that it results in dense matrices, which lead to high memory requirement ($\mathcal{O}(N^2)$, if N is the total number of DoF) and computational costs. Therefore, the simulation of realistic problems is limited by the number of DoF. Several fast BEMs have been developed to improve the computational efficiency. In particular, the hierarchical matrix ($\mathcal{H}$-matrix) technique [3] permits to approximate the fully-populated BEM matrix by a data-sparse one and to accelerate the classical matrix/vector product, in order to define fast iterative solvers for elastodynamic problems. Benedetti et al. [1, 4] have exploited this solution strategy in the context of 3D elastodynamic crack problems and for anisotropic elastodynamic media, while Coulier et al. [2] have used an $\mathcal{H}$-matrix based iterative BEM solver for elastodynamics based on Green's function for a horizontally layered half-space. We propose a fast $\mathcal{H}$-matrix based direct BEM solver for 3D elastodynamic problems. If the reduction of storage cost is due to low-rank approximations computed by the innovative Vector Adaptive Cross Approximation (VACA), the reduction of the computational time to solve the system comes from a LU-factorization computed by using $\mathcal{H}$-matrix arithmetic. The numerical efficiency and accuracy of the method are assessed on the basis of numerical results obtained for benchmarks problems with explicit solutions. In particular, a numerical study of the efficiency of low-rank approximations when the frequency is increased is presented. The efficiency of the method is also illustrated to simulate an elastic half-space with or without topographic irregularities.

References

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