Optimizing some 3-stage W-methods for the time integration of PDEs

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Abstract. The optimization of some W-methods [7] for the time integration of time-dependent PDEs in several spatial variables is considered. In [2, Theorem 1] several three-parametric families of three-stage W-methods for the integration of IVPs in ODEs were studied. Besides, the optimization of several specific methods for PDEs when the Approximate Matrix Factorization Splitting (AMF) [3, 4] is used to define the approximate Jacobian matrix ($W \approx f_y(y_n)$) was carried out. Also, some convergence and stability properties were presented [2]. The derived methods were optimized on the base that the underlying explicit Runge-Kutta method is the one having the largest Monotonicity interval among the thee-stage order three Runge-Kutta methods [1]. Here, we propose an optimization of the methods by imposing some additional order condition [6] to keep order three for parabolic PDE problems [5] but at the price of reducing substantially the length of the nonlinear Monotonicity interval of the underlying explicit Runge-Kutta method.

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