Analysis and Numerical Treatment of Singular BVPs in ODEs with Nonsmooth Inhomogeneities

E.B. Weinmüller (Vienna University of Technology, Vienna, Austria)

joint work with
J. Burkotova and I. Rachunkova (Palacky University Olomouc, Czech Republic)

We study BVPs for systems of ODEs with singular points in the differential operator. Typically, such problems have the form

$$z'(t) = \frac{M(t)}{t} z(t) + f(t, z(t)), \;\; t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where in general, matrices B_0 and B_1 are subject to certain restrictions in order for the BVP to be well-posed. Here, we focus on the linear case with a variable coefficient matrix $M(t)$, where the inhomogeneity f is nonsmooth, $f(t) = g(t)/t$. We first deal with the analytical properties of the problem – existence and uniqueness of solutions – with the focus on the structure of BCs which are necessary and sufficient for the solution z to be at least continuous, $z \in C[0, 1]$ [1].
To approximately solve the problem, we apply polynomial collocation and discuss its convergence properties. It turns out that the collocation retains its high convergence order (stage order) even in the case when of singularities are present, provided that the analytical solution is sufficiently smooth [2]. We illustrate the theory by means of numerical experiments carried out using the MATLAB code `bvpsuite` [3].

References

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[3] G. Kitzhofer, O. Koch, G. Pulverer, Ch. Simon, E.B. Weinmüller, The New MATLAB Code `bvpsuite` for the Solution of Singular Implicit BVPs. *JNAIAM J. Numer. Anal. Indust. Appl. Math., 5* (2010), 113-134.