BVP in Differential-Algebraic Equations and their numerical solution using collocation

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We consider nonlinear BVPs in DAEs with properly stated leading term. Such problems can be written as

$$f((Dx)'(t), x(t), t) = 0, \quad t \in (a, b), \quad g(x(a), x(b)) = 0,$$

where typically, the partial Jacobian of $f(y, x, t)$, $f_y(y, x, t)$, is singular everywhere. First, we prcisely describe the problem setting and point out the main differences compared with the standard BVPs in ODEs. Then, we introduce and discuss the following notions describing the problem and known from the ODE context: well-posed BVP, accurately stated boundary condition, locally unique, and isolated solution. Finally, we describe the relations between those notions.

Next, the convergence behavior of the collocation method applied to an index 1 DAE system in its original form is discussed in both cases of a regular and singular inherent ODE system. While for the index 1 problems collocation turns out to be a robust and fast convergent method, it can fail for higher index case. By means of examples, we illustrate the limits of applicability of collocation in this case.

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